The Impact of Consumer’s Bounded Rationality on Retailer’s Price and Location Decisions

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Abstract

As is the case with most OR models, the original maximal covering location problem with price decision (PrMAXCOV) assumes perfect rationality on the part of the consumer. This is clearly contradicted by the evidence found in marketing literature. Starting from this experimental evidence, we illustrate through the PrMAXCOV how these mathematical models can be adapted in order to better reflect the consumer’s bounded rationality that appears in real life.

First, we examine the impact of rounded pricing on the solution of the model. Our results show that simply rounding the solution of a given problem may not always be advisable. In some cases it is advantageous to restrict the possible solutions to rounded values in advance. Based on the rounded optimal solution, we show that expected revenue is actually an underestimation due to the drop-off mechanism. Secondly, we illustrate that when retailers limit their prices to predetermined ranges, this will almost always result in a loss of revenue. Finally, we show that when the model considers the fact that certain customers pay little attention to the price, firms will generally charge higher prices and locate closer to these inattentive and/or uninformed customers.

Keywords: Location, Marketing, Pricing, Decision making

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1 Introduction

When entering a competitive market, a firm has to make several crucial choices. These decisions relate to all possible aspects of the enterprise and the products it provides. Combined, these decisions make up the firm’s strategy towards achieving its objectives. In most companies, this means maximizing shareholder value and/or profits.

In marketing literature, the firm’s strategic decisions are generally grouped into four categories, commonly known as the four P’s: product, price, place and promotion. In each of these categories a firm must position itself vis-a-vis the competition. The model used in this article assumes the product to be homogeneous, thus ruling out any differentiation within that category of the marketing mix. We also ignore promotional activities. This leaves two important factors to be determined, namely place (location) and price.

When deciding on location and pricing strategy, it is important for a firm to consider the manner in which consumers will react to them. This will, for a large part, depend on the decision making process employed by these consumers. Most significantly, a firm must determine what information is used by its customers and how this information is processed.

Most models within the field of locational analysis assume that the customers are fully informed and perfectly rational. For a detailed overview of consumer behavior modeling, see Vanhaverbeke [1]. This means that they do not only have access to all the necessary information, they also process it in a fully rational manner. However, empirical evidence suggests that “consumer rationality is a property of the researcher’s model rather than the consumer” [2].

As a solution towards reducing the discrepancy between consumer behavior in location analysis models and the behavior as it is described in marketing literature, we propose to implement some degree of bounded rationality. Simon [3], who coined the phrase of bounded rationality, extended the “rational choice theory by including decision-makers that have limited knowledge and finite cognitive abilities”. In doing so, he “discards the assumption that customers have perfect information and, for that matter, neither have the information they need nor know what information they need” [4].

To illustrate the impact that bounded rationality on the consumer’s part can have on a retailer’s location and price decisions, a model is required in which the optimal choices for both these variables are decided upon by the retailer. Formal models with the sole objective of determining a firm’s optimal location “have been studied in various forms for hundreds of years” [5]. Their emergence is most often linked with the seminal paper titled “Stability in Competition” [6]. In that article Hotelling established the “Principle of Minimal Differentiation” for a duopoly on a linear market, proving that in equilibrium both firms will locate at the middle of the market. This result was later shown to be a consequence of the stringent assumption used in the model. D’Aspremont et al. [7] even found the opposite result when slightly altering the assumptions of the original model. Nevertheless, he “set the foundations
of what is today the burgeoning field of competitive location” [8]. An overview of recent work within the broad field of locational analysis has been assembled by ReVelle et al. [5].

However, most of these competitive location models did not attribute the retailer with the ability to determine a preferred price level. An initial price-location model was developed by Serra and ReVelle [8], who extended the maximum capture model [9] with a price decision, using a heuristic method to solve the problem. An alternative approach has been presented by Plastria and Vanhaverbeke [10]. Their maximal covering location problem with price decision (PrMAXCOV) determines the optimal location of the firm’s outlets and the price that must be charged in order to generate maximal revenue. Contrary to Serra and ReVelle [8], they presented an exact solution method.

The PrMAXCOV model will be used here to illustrate the effects that consumer’s bounded rationality can have on the retailer’s price and location decisions. The model and the solution methods as developed by Plastria and Vanhaverbeke will be presented in section 2. This is followed by a comparison between the consumer behavior as it was assumed in the model and the behavior that is described in marketing literature, based on empirical evidence. In an attempt to lessen the difference between these two approaches, we formulate some consumer inspired variations to the model in section 3 and examine their impact on the optimal price levels, outlet locations and the retailer’s revenue. These will be complemented by two practical restrictions that retailer may be faced with and which also may have considerable influence on the retailer’s price and location decisions. Some concluding remarks are presented in section 4.

2 Consumer rationality in the PrMAXCOV

The PrMAXCOV by Plastria and Vanhaverbeke [10] determines the optimal locations and optimal price for a firm that is entering a market where competitors are already present. The decisions regarding location of outlets and price level made by the firm are based on the behavior that consumers are assumed to display. As is the case for most models within the field of locational analysis, this behavior is regarded as perfectly rational and fully informed.

When a firm is a wholesaler, perfect rationality can be a reasonable assumption with regard to its customers. A wholesale company supplies other companies, who often employ professional buyers. These purchasers (aim to) use all the information available to them and often use tools to process this information, such as spreadsheets. This enables them to make fully informed and rational decisions. However, retail consumer behavior is often not as rational as assumed in most models. Among the most common explanations are bounded rationality and imperfect information. This raises the question on how consumer behavior as perceived in marketing literature relates to the behavior as modeled in locational analysis. A comparison between these two approaches is the subject of this section, illustrated by the
PrMAXCOV.

2.1 The model

Extending the classic Maximal Covering Location Problem by Church and ReVelle [11] with a price decision, Plastria and Vanhaverbeke chose to employ to a large extent the same notations. A firm entering a competitive market will locate a fixed, predetermined number of outlets at a finite number of locations (set \( J \)), given the fixed locations of the competing stores (set \( C \)). As both the classic maximal covering model and the PrMAXCOV are discrete models, customers (set \( I \)) are represented by demand points. As mentioned by the authors, these points are often the result of an aggregation process done beforehand, as described in an earlier article [12]. For a survey on aggregation methods and errors in location models, see Francis et al. [13].

Besides the obvious difference in objective functions, there is one important point of contrast between the revenue maximizing model and the classic model. Church and ReVelle [11] maximize the covered demand, with the added restriction that “no demand point will be farther than the maximal service distance from a facility”. This reflects the fact that, for inessential goods, consumers will only travel a predetermined maximal distance (\( d_{\text{max}} \)) in order to make a purchase. Plastria and Vanhaverbeke consider the case of essential goods, who “will always be purchased, no matter how far the customer must travel to acquire the good” [10]. The possibility of introducing \( d_{\text{max}} \) in their model is mentioned, but not implemented.

The objective of the PrMAXCOV is to maximize revenue, rather than covered demand. This is achieved by introducing the mill price (\( p \)) in the objective function of the model, shown in equation (1), and multiplying it with the demand covered.

\[
\prod^* = \max \sum_{i \in I} q_i x_i p
\]

The covered demand is determined by multiplying the amount of demand represented by each customer \( q_i \) by a “covering variable \( x_i \), taking the value 1 when the customer \( i \) is covered by at least one of the outlets of the new firm or remains 0 if not” [10].

Customers will not make their buying decision based only on the mill price at the outlet, but rather on the total purchasing cost. This total cost is composed of the “price \( p \) charged by the new player and the transportation cost \( t \) over the distance \( d_{ij} \) traveled by customer \( i \) to outlet \( j \)” [10]. Customers are assumed to show novelty behavior, which implies that “in case of a tie, the new facility gets all demand” [14]. A tie can occur when, by accident, a customer’s total purchasing cost is equal for the existing and the new player. Plastria and Vanhaverbeke [10] subsequently defined that “customer \( i \) will patronize a facility \( j \in J \) if the sum of the price charged by the existing players plus his transportation cost to the existing
player’s closest outlet is strictly higher than the total price at the new outlet” [10]. All this is stated in the following inequality:

\[ p + td_{ij} \leq \min_{c \in C} (p_c + td_{ic}) \quad \forall i \in I, j \in J \quad (2) \]

This leads to the definition of the so-called critical price: “the minimum of the total prices of all existing player outlets for customer \( i \), minus the transportation costs for the distance from customer \( i \) to the new outlet \( j \)” [10]. This is called the critical price because if the price \( p \) charged by the new firm undercuts \( p_{ij} \), customer \( i \) will patronize a new outlet at \( j \).

\[ p_{ij} = \min_{c \in C} (p_c + td_{ic}) - td_{ij} \quad \forall i \in I, j \in J \quad (3) \]

Based on the critical price, Plastria and Vanhaverbeke [10] also defined the patronizing set \( N_i(p) \) as “containing all facilities for which the critical price \( p_{ij} \) is at least \( p \)”: \[ N_i(p) = \{ j \in J : p_{ij} \geq p \} \quad \forall i \in I \quad (4) \]

The following mathematical formulation of the complete model was defined:

\[
\text{max } \sum_{i \in I} q_i x_i p \\
\text{s.t. } \sum_{j \in N_i(p)} y_j \geq x_i \quad \forall i \in I \\
\sum_{j \in J} y_j = B \\
0 \leq x_i \leq 1 \quad \forall i \in I \\
y_j \in \{0, 1\} \quad \forall j \in J \\
p \geq p_{\text{min}}
\]

Equation (5) is the objective function, which was already analyzed above. Equation (6) expresses the so-called covering constraint. It defines that “at least one facility in the set \( N_i(p) \) must be opened for customer \( i \) to be covered by the new firm, under price setting \( p \)” [10]. The location variable \( (y_j) \) takes value 1 if a new outlet is opened at location \( j \) and 0 otherwise, which is defined in equation (9).

One could expect the definition of the covering variable \( (x_i) \) in equation (8) to have a similar expression as the location variable \( (y_j) \). Unlike the location variable, the covering variable is not restricted to binary values, but rather “continuous and bounded between 0 and 1” [15]. This is due to the fact that the optimization process will push the variable to its extreme values 0 or 1. From equation (9) it can be deduced that the constraining values in the covering constraint (6) will either be 0 or at least 1. As the covering variable is part of the
objective function it will always be maximized, resulting in a *de facto* binary value. In practice it appears that when using commercial software “any reduction in the number of binary variables may allow to increase the problem size without reaching the limits of tractability” [15].

Equation (7) expresses the budget constraint. “A new firm is limited to a number $B$ of new facilities to open” [10]. This can be easily modified into a monetary budget constraint if a fixed cost is attributed to each facility. Finally, a minimum price is defined in equation (10), which may not be undercut. It expresses the fact that costs must be covered by the price and “operating with loss is not allowed” [10]. Plastria and Vanhaverbeke assumed that all firms have the same cost structure, which allowed them to make abstraction of the variable cost. The advantage of using this assumption is that, given the identical cost structure of all outlets, revenue maximization corresponds to profit maximization.

As mentioned in the introductory section, Plastria and Vanhaverbeke have developed an exact solution method for determining the maximal revenue for the PrMAXCOV. They declare in their paper: “this problem is a special case of the heuristically solved problem “PMAXCAP” by Serra and ReVelle” [10]. The difference lies in the fact that, unlike Serra and ReVelle, they do not take into account demand elasticity.

The solution method was developed in two main steps. First, they determine a full enumeration method, which is only applicable for small size problems due to the limited capacity of commercially available software. This is followed by an intelligent enumeration method, using “some characteristics of the covering problem to help narrow down the solution space and avoid full enumeration” [10].

Starting with the full enumeration method, they use the fact that a maximal covering problem can be defined for any fixed $p$. Examining the relationships between the covering problems for different values of $p$, Plastria and Vanhaverbeke reveal a finite domination property of the set of all feasible critical prices of the deduced revenue maximization model. This is expressed in properties 1 through 4 in their article [10].

A list $P$ is calculated as the set of all critical prices $p_{ij}$, under the initial assumption that they are all pairwise different and greater or equal to the set minimum price. Given that abstraction was made of the variable cost, the minimum price is held constant at 0.

$$P = \{p_{ij} \geq p_{\text{min}} | (i \in I, j \in J)\}$$  \hspace{1cm} (11)
Then for every $p \in P$, they define $CP(p)$ as the maximal covering problem at $p$, for which $CP^*(p)$ is the optimal value:

$$\begin{align*}
\max \quad & \sum_{i \in I} q_i x_i \\
\text{s.t.} \quad & \sum_{j \in N_i(p)} y_j \geq x_i \quad \forall i \in I \\
& \sum_{j \in J} y_j = B \\
& 0 \leq x_i \leq 1 \quad \forall i \in I \\
& y_j \in \{0, 1\} \quad \forall j \in J
\end{align*}$$

When comparing the maximal covered demand for different prices, it is shown that “if the new firm is able to cover a certain amount of demand with price $p'$, then at least the same amount will be covered with a price $p$ smaller than $p'$” [10]. The formal proof can be found in the first three properties developed by Plastria and Vanhaverbeke. Consequently, their fourth property showed that in order to maximize revenue, only the $p \in P$ need to be examined, thus limiting the number of potential optimal solutions.

As mentioned above, using the full enumeration of all $p \in P$ allows the calculation of the exact optimal solution of the problem, but even relatively small problems quickly reach the computational limits of commercially available software. This is why Plastria and Vanhaverbeke developed a more intelligent enumeration method, based on properties that allow the further reduction of the number of prices that needs to be considered.

The intelligent enumeration method for solving the PrMAXCOV is established in their properties 5 through 9. These rely on the fact that the list $P$ is sorted increasingly. Plastria and Vanhaverbeke describe property 5 as a “bounding rule to predict from the results obtained for one price, that some of the following prices in $P$ should not be considered” [10]. This is followed by two “rules which allow to predict that the optimal solution for one price in $P$ remains optimal for the next price in $P$”, which are proven in properties 6 and 7.

The final two properties deal with the case of non-unique critical prices. Depending on the problem, it may occur that “for two different pairs of $(i \in I, j \in J)$ the values of the critical prices coincide” [10]. As these properties will not be used here, we will not go into detail on them.

### 2.2 Are consumers really rational?

As discussed above, Plastria and Vanhaverbeke state in their article that “customer $i$ will patronize a facility at $j \in J$ if its total price is the lowest in the market; in other words, if the sum of the price charged by the existing players plus his transportation cost to the existing player’s closest outlet is strictly higher than the total price at the new outlet” [10], which is expressed by the inequality (2).
This statement entails several assumptions concerning the customer decision making process. In what follows, the point of view of the customer will be taken, not that of the firm. The locations and price are determined by the firm, and the customers use the available information to make their purchase decision. The goal of the firm is of course to make sure that as many consumers as possible will favor its locations and products when they evaluate the different players in the market. Remember that the following assumptions were made by Plastria and Vanhaverbeke: essential goods, a homogeneous product, novelty behavior and no demand elasticity.

Figure 1 shows the separate steps of the decision making process as it is assumed in the PrMAXCOV. Firstly, customers will need to have access to a lot of information in order to evaluate all outlets in the market, as equation (2) includes many parameters. This means that in order to make a decision based on the criteria in equation (2), customers must acquire and have access to all of this information. This includes the transportation cost, the new firm’s price, the distance to the new firm’s nearest outlets, the competitors prices, and the distance to the competition’s outlets.

When the consumer obtains all information, he will neatly compute the total costs faced when shopping at a given outlet. This means determining the cost to get there and adding the mill price, charged at the outlet. Finally, the consumer decides at which outlet it is cheapest for him/her to buy the product.

When modeling the behavior of firms, rationality and access to information are plausible assumptions. When firms decide where they will locate their new outlets and which price to charge, they aim to acquire as much relevant information as possible. This enables trained personnel to process this information in order to make the best possible choices. In such a decision making process, models such as the PrMAXCOV [10] can provide considerable assistance in determining the best options for the firm.

However, the firm that wants to enter a new market will evidently sell its products to any customer who has demand for the product. The question raised here is how this rational, informed behavior relates to consumer behavior as described in marketing literature? It appears that when a firm is active in a business-to-customer or retail market, customers are often far from rational. In what follows, several examples from marketing literature will be
provided where empirical findings indicate that consumer behavior differs significantly from the process that was described in figure 1.

Shugan states it like this: “consumer rationality is a property of the researcher rather than the consumer” [2]. Although his remark is directed towards marketing research, it also applies to locational analysis. He predicts that “a clear and fundamental understanding of consumer behavior should help us more accurately predict consumer reactions to marketing interventions”. Translating this statement to the models that were discussed above, this means that the goal must be to model consumer behavior as closely to reality as possible. This will in turn increase the predictive power of the models in terms of the customers reactions to the location and price decision of the firm.

In the following sections, we present some findings from marketing literature. These empirical results, illustrating the consumer’s bounded rationality, inspired the modifications that are discussed in section 3.

2.2.1 Information seeking and knowledge

The first step in the consumer decision making process in the PrMAXCOV consists of gathering the relevant information. To make the trade-off between the different outlets in the market, the customer must acquire all prices, transportation costs and distances between his location and the outlets. Although consumers generally tend to be not that well informed, some variation may exist.

In an early attempt to distinguish the different types of consumer that may exist, in terms of information seeking, Claxton et al. [16] developed a numerical taxonomy. Considering only the purchase of durable goods, their taxonomic approach made it possible “to classify buyers into distinctive groups in terms of their prepurchase search patterns”. They found three measures (number of information sources used, total visits to stores, and deliberation time) that allowed them to identify three general clusters of customers based on the information seeking behavior. These were labeled “thorough (store intense), thorough (balanced) and non-thorough”, which were further subdivided into “subclusters varying in deliberation time” [16]. Their results provide clear evidence that consumers can be divided into segments based on information seeking behavior.

More recently, Schmidt and Spreng [17] suggest that many variables influence pre-purchase information search, but these effects are mediated by four variables: ability, motivation, cost and benefit. They integrated a psychological framework concentrating on personal aspects of the consumer with an economic framework that focuses on monetary costs and benefits. Based on these four variables, a consumer’s information search effort can be predicted and thus the information that will be used in the eventual purchase decision.

With the aim of finding a basis for pricing differentiation in the retail grocery market, Berné et al. [18] performed a cluster analysis to identify consumer profile based on their
price-information-seeking behavior. They found “two clearly separated clusters: high and low intensity price-information seekers”[18]. This means that, based on their survey, a number of customers will spend much time and possibly money in the search for price information, while others will be satisfied with the information that is readily available.

Following this initial article, Berné et al. [19] stated two years later that “there has been an overestimation of the proportion of consumers who actively search for prices”. They empirically examined factors which may influence consumer information-seeking behavior. Their results showed that human ability holds “the greatest predictive power to explain the tendency for price comparison”. This confirms the findings of Schmidt and Spreng [17]. According to the study, “overall market knowledge and previous investment in price search are the principal predictors of price search”. Also analyzing demographic characteristics, Berné et al. found that “the higher the age of the consumer, the greater his/her tendency to compare prices”[19].

These findings indicate that it would be advisable to be able to distinguish different types of customers, based on these characteristics. Yet, in the PrMAXCOV it was assumed that all customers react in the same way to the price and locations of the firm and that they use the same amount of information.

A factor not considered by Berné et al. was the type of product. An alternative study by Aalto-Setälä and Raijas investigated “the degree of consumers’ price-consciousness, taking into account the price dispersion of the grocery products that are at least physically homogeneous” [20]. This relates more closely to the assumptions of the PrMAXCOV [10].

Based on empirical data containing the “price observations of eight grocery items in 82 grocery stores in Finland” and the results of the interview of 1000 Finnish consumers by phone, Aalto-Setälä and Raijas conclude that “the consumers were very well informed about the price of homogeneous products such as milk and sugar” [20].

Although the study by Aalto-Setälä and Raijas is limited to Finnish consumers and grocery products, it may be seen as supportive of the assumptions on consumer price knowledge in the PrMAXCOV. These, or any other, studies did not include consumer knowledge on the transportation costs and the distance to outlets. One can never really claim to have searched the entire literature on consumer behavior, but so far we have not found any studies in this particular topic. Yet, this information was also required to make the fully informed purchasing decision that was assumed in the PrMAXCOV. This could be an interesting avenue for further research.

Concluding, when working under the assumption of a homogeneous product, consumers have relatively good price knowledge. We make abstraction of the consumer knowledge of transportation costs. In the following sections, we will assume that consumers who are well informed on prices also have access to additional information to make an informed decision. We will now turn to the next part of the purchase decision, where this information is processed.
2.2.2 Information processing

Assuming that consumers have access to all the information they require, the question remains how much of this information they will use in their decision making process. Stiving and Winer state that “when consumers evaluate the price of a good, they may consider the whole price, as assumed by most marketeers and economists, or they may use some heuristic to simplify the task” [21]. Plastria and Vanhaverbeke [10] clearly follow the former reasoning, rather than the heuristic approach. The comparison between both approaches will be presented here, followed by some computational results in section 3.1.

When looking at the results in the original article [10], the price ending digits were divided more or less equally between all digits from 0 to 9. However, this is not the case in many retail outlets. The vast majority of price-endings are of the numbers 0, 5 or 9. Schindler and Kirby [22] empirically found that, out of 1,415 advertised prices, 27.2% ended in 0; 18.5% ended in 5 and 30.7% had 9 as the rightmost digit. This means that those three digits made up a vast majority of 76.4% price endings. Similar, older studies are provided by Gendall et al. [23].

The practice of setting prices just below the even level is most often referred to as “odd pricing” [21, 22, 23]. Strictly speaking, odd pricing can refer to any price that is not a round price, or “prices ending in one of the odd digits 1, 3, 5, 7 or 9” [24], but here it will refer to “a price set slightly below a round number” [25].

For a long time “the results of attempts to validate the assumption that odd prices lead to greater than expected demand have been mixed and inconclusive” [23]. Several theories have been developed over the years in order to explain the positive effect that odd pricing is believed to have on sales. These theories aim to provide a conclusive rationale for the disproportional use of “just below prices” [25] in retail. Figure 2, developed by Stiving and Winer [21] shows the different explanations for price endings. One slight difference with their original schematic is our addition of the monetary unit to the operations category.

The two original explanations within the operations category, presented by Stiving and Winer [21], were not examined in great detail. One possible reason for charging prices just below the even level, given by Schindler and Kirby [22], is “because most customers paid in even-dollar amounts, a 9-ending price would oblige clerks to use the cash register to make change and thus reduce their opportunity to pocket the payment”.

We pose that monetary unit should be added as a third explanation within the operations category. The price endings available to a firm are restricted to multiples of the smallest denomination in circulation in the countries in which it is active. This is illustrated by the case of Finland, where the one and two eurocent coins have been taken out of circulation, although they remain legal tender. This obliged retailers to make sure that the bottom line on each bill is a multiple of five cents, as individual products may still have prices that are multiples of one cent. Under our assumption of only one product, this implies that the price
would have to be a multiple of five cents.

Figure 2: Explanations for price endings based on [21], with the addition of the category monetary unit.

The image effects, featured in the right hand of figure 2, are also not relevant in the context of the PrMAXCOV. This group of theories assumes that the price, or the form of the price, projects a certain image of the product to the customer. Different explanations have been provided, e.g. that “odd prices indicate low-quality merchandise” [21]. A more detailed analysis is provided by Schindler and Kibarian [26], who also found that a “99 price ending decreased the viewer’s perception of the quality of the advertised item, as well as of the general merchandise quality and the retailer’s classiness”. Here, as we continue to work under the assumption of homogeneous products, no differentiation is made based of the quality of the product.

The most interesting price ending effects, from the viewpoint of the PrMAXCOV, are the “level effects” [21, 24, 25]. These *level effects*, “also known as underestimation effects, refer to the behaviors or underlying processes that cause a consumer to distort their perception of the price” [21]. The underlying idea in these theories is that consumers do not include the price he/she reads correctly in his/her memory, which is often referred to as the “drop-off mechanism” [27].

Several explanations have been presented in the literature: rounding down, comparing prices left to right and the limited memory capacity of the consumer. Stiving and Winer already noted that “if consumers actually round down prices, firms would have a great incentive to use just-below prices, providing a explanation for the observed price endings” [21]. This means that retailers would be best off to maximize the rightmost digits of prices, as consumers do not take them into account in their decision making process.

Bizer presented a study in which the goal was to “directly test whether consumers do in-
deed tend to drop off rightmost numerical digits” [27]. His experiment consisted of giving his test subjects a hypothetical budget ($73) and they were asked to estimate the number of items that could be purchased with this budget. “The ending digits of the price was varied—half of the time it was a 99-ending price (e.g., $2.99, $4.99) and the other half of the time it was the 00-ending price one penny higher (e.g., $3.00, $5.00)” [27]. He found that “respondents estimated that significantly more items with 99-ending prices could be purchased for a given amount of money than items with the 00-ending prices only one cent higher”, which provides “direct evidence for the existence of the drop-off mechanism in price information processing”. Other empirical studies were performed by Manning and Sprott [25] and Guéguen et al. [28], who largely confirm Bizer’s results in different settings.

When considering the rationale of the drop-off mechanism, it can be said that the PrMAX-COV underestimates the revenue that a firm may generate when entering a market. The model assumes that the consumer considers the whole price when he/she makes his/her purchasing decision. The analysis made in marketing literature, discussed above, indicates the opposite. Consumers generally compare price from left to right, even ignoring the right most digits entirely.

Assuming that consumers indeed ignore the right hand digits (usually the decimals), retailers should implement this form of irrational behavior in their location decision. For the PrMAXCOV, this would imply determining the demand covered based on rounded prices, and subsequently maximizing the decimals (to .99) as customers do not include them in the decision making process. In section 3.1, some computational results will be presented, showing the impact of the drop-off mechanism on the firm’s revenue.

3 A consumer based approach to the PrMAXCOV

From the previous section, it may be clear that there exist considerable discrepancies between consumer behavior as it is assumed in marketing literature and in location analysis models. We will now present three modifications to the original model, addressing some of these concerns. Together with implementing a more consumer oriented approach, we provide an answer to a question raised by Plastria and Vanhaverbeke:

“The price setting in a retail outlet does not allow for the very precise figures we propose here. We are interested to see the impact of rounded prices on the solution of the model. Another option we consider is the use of a selective price choice heuristic. Imagine the retailer to be interested in certain ranges of price, what would be the effect on the optimal model solution?” [10]

This question can be divided into two separate issues: the impact of rounded prices and the effect of price ranges on the solution of the model. The first issue regarding rounded
pricing will be examined in section 3.1, which in turn allows the implementation of the drop-off mechanism. The impact of price ranges on the solution of the problem is subsequently presented in section 3.2. Finally, in section 3.3 an alteration to the definition of the covering sets is discussed, allowing for the differentiation between different consumer types.

The results presented below were generated using the procedure and model that was developed by Plastria and Vanhaverbeke. They wrote the program in AMPL (A Modeling Language for Mathematical Programming, Version 20021038) and solved using ILOG CPLEX 9.1. One slight difference is that for this paper version 20080925 was used for AMPL and version 11.0 for CPLEX to calculate the results. The hardware used is the same as in the original article, namely a Intel Xeon CPU 3.4GHz, with 2GB RAM, running Windows XP Profesional SP2. To allow for comparison of the results, we used for the most part the same data that was used in the original article. Throughout what follows, the results for the small case and the large case [10] are presented to allow for a smooth comparison with the original results. The test data are available at http://homepages.vub.ac.be/~l vhaverb/testdata.html.

As described in [10], the small case considers the region of the Belgian capital (Brussels). There are 33 customers (I), 6 existing competitors (C) whose price varies randomly between 55 and 67 and there are 10 locations at which the firm can potentially locate (J). The distances were calculated on the road network and the demand was randomly generated with values between 20 and 39. The large case was completely artificially generated, with 1,000 customers whose x- and y-coordinates are random between 0 and 100 as well as their demand that lies between 20 and 50. There are 100 potential locations for the new firm to open an outlet with 20 competitors already present in the market. In the large case, the distances are Euclidean.

3.1 Rounded pricing and drop-off mechanism

From the computational results presented in [10], it might appear that there is no problem with the number of decimals from a practical point of view. The tables containing the results for the three datasets only show two decimals for the optimal prices. However, these are not the exact optimal solutions to the problem. In reality, the exact prices often had up to nine decimals.

It may be clear that when dealing with the exact solution of the model, there arises a problem in terms of practicality. When a firm decides to implement the optimal pricing policy that the model calculated, an obvious restriction exists on the number of decimals. All prices are bound to the currency of the country in which the firm wishes to locate itself. For most currencies the smallest denomination is one cent, representing one hundredth of the base value. This means that in practice prices are limited to two decimals, as all prices must be multiples of the smallest denomination in circulation. In this respect, Finland is an even more restrictive case, in that all prices must be multiples of five cents.
Intuitively, calculating the exact optimal solution and rounding the optimal price to the nearest feasible value appears to be the simplest approach in dealing with this issue. This is also the method used by Plastria and Vanhaverbeke. An alternative approach will be proposed here, namely ensuring in advance that the solution calculated by the model is rounded and thus practically feasible.

Ensuring that the solution of the model is a rounded value can be done by an alteration in the first step of the solution procedure of the model. Depending on the smallest denomination, available for a given problem, the list $P$ is replaced by an artificial list $P_a$, consisting of multiples of this smallest denomination ($a$). It is also manually ensured that the list $P_a$ consists solely of unique prices, making properties 8 and 9 from the PrMAXCOV redundant.

As the list $P_a$ is artificial, independent of the problem under consideration, the question arises how long the list will have be, in order to ensure a maximal solution. The answer is as follows: the maximal price $p_a \in P_a$ must be the highest multiple of the smallest denomination ($a$) which is still less or equal to the maximal critical price ($\text{max} P$). This can be explained by the fact that any price larger than the $\text{max} P$ will not undercut the existing minimum total price $p_{ij}$ for any customer $i$ at any new outlet $j$, so no customer will patronize any outlet situated at any location $j \in J$. In other words, no demand will be covered, so this can never be an optimal solution to the problem. This means that the list $P_a$ will be finite and will comprise of $\text{max} P/a$ prices.

In the presentation of the model in section 2.1 it was mentioned that two solution methods were developed for the PrMAXCOV, a full enumeration method and a faster, intelligent method. As the latter is based on the properties of the list $P$, it cannot be used when working with $P_a$. Although the fifth property developed by Plastria and Vanhaverbeke remains valid, the final four properties do not. They rely on the relation between the covering sets for two consecutive critical prices $p_{ij} \in P$, which does not exist when working with the artificial prices $p_a$.

Now we can turn to the question how the solution of the unrestricted model ($\Pi^*$) will relate to the solution using the artificial, rounded prices ($\Pi^*_a$). Generally, it can be expected that adding restrictions to a model will decrease, rather than increase, the optimal value of any problem. Plastria and Vanhaverbeke noted that they “only need to examine prices in $P$, i.e. of critical price type $p_{ij}$” [10], meaning that only these prices can result in an unrestricted optimal solution. This implies that no price $p_a \in P_a$ will result in higher revenue for the firm.

The interesting question is whether or not the optimal price determined using the list $P_a$ will be the same as rounding the unrestricted optimal price to the nearest multiple of the smallest denomination ($a$). Table 1 makes this comparison with respect to the large case used by Plastria and Vanhaverbeke.

Table 1 shows the optimal price and revenue for both the unrestricted calculation as well as for the model using the rounded pricelist $P_a$. It may be clear that the unrestricted solutions
Table 1: Comparison between the optimal price and revenue for the unrestricted model and the restricted solution for three values of \( a \). Several values of the variable \( B \) are presented.

<table>
<thead>
<tr>
<th>Budget</th>
<th>( p^* )</th>
<th>( p_{0.01} )</th>
<th>( p_{0.05} )</th>
<th>( p_1 )</th>
<th>( \Pi^* )</th>
<th>( \Pi_{0.01} )</th>
<th>( \Pi_{0.05} )</th>
<th>( \Pi_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>10.8194</td>
<td>10.81</td>
<td>10.35</td>
<td>10</td>
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<td>21,135.21</td>
<td>21,121.37</td>
<td>20,899.84</td>
</tr>
<tr>
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<td>10.35</td>
<td>10.35</td>
<td>10</td>
<td>38,187.32</td>
<td>38,160.80</td>
<td>38,160.80</td>
<td>37,576.38</td>
</tr>
<tr>
<td>3</td>
<td>4.9742</td>
<td>4.97</td>
<td>5.00</td>
<td>5</td>
<td>54,888.86</td>
<td>54,842.64</td>
<td>54,638.28</td>
<td>54,638.28</td>
</tr>
<tr>
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<td>4.97</td>
<td>5.00</td>
<td>5</td>
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<td>71,348.48</td>
<td>71,243.74</td>
<td>71,243.74</td>
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<td>5.05</td>
<td>5.05</td>
<td>5</td>
<td>84,369.53</td>
<td>84,240.41</td>
<td>84,240.41</td>
<td>84,122.66</td>
</tr>
<tr>
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<td>5.00</td>
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<td>5.05</td>
<td>5</td>
<td>106,814.30</td>
<td>106,650.84</td>
<td>106,650.84</td>
<td>106,378.10</td>
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<tr>
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<td>5.03</td>
<td>5.05</td>
<td>5</td>
<td>116,318.73</td>
<td>116,153.98</td>
<td>116,140.72</td>
<td>115,685.66</td>
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<td>5.05</td>
<td>5</td>
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<td>123,785.69</td>
<td>123,785.69</td>
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<td>129,560.53</td>
<td>129,560.53</td>
<td>128,895.02</td>
</tr>
</tbody>
</table>

\( p^* \) and \( \Pi^* \) are the results that were already determined by Plastria and Vanhaverbeke [10]. Note that the columns \( p_{0.05}^* \) and \( \Pi_{0.05}^* \) are an illustration of the case of Finland, with the smallest denomination being five eurocent.

It appears that there exists a negative relation between amount of the smallest denomination and revenue. Obviously, when the solution for a certain value of \( a \) (e.g. 0.01) is possible for a larger value of \( a \) (e.g. 0.05), the revenue will be the same. When this is not the case, an increased value of \( a \) will, ceteris paribus, result in a reduced revenue as the set \( P_a \) from which the solution can be drawn becomes smaller. We know for certain that \( p_{0.05}^* \in P_{0.01}^* \), while on the other hand it is only by coincidence that \( p_{0.01}^* \in P_{0.05}^* \) may occur.

Looking at the price, in most cases the \( p_a \) is the closest rounded value of the unrestricted optimal price \( p^* \). One exception is the case of \( B = 1 \), where it would be expected that \( p_{0.05}^* = 10.80 \), but calculations showed that \( p_{0.05}^* = 10.35 \). This implies that, instead of using the unrestricted model and rounding the optimal solution, it would be advantageous for a firm to use the restricted model.

Of course, the PrMAXCOV not only determines the optimal price \( p \), but also the optimal locations at which the firm must open (an) outlet(s). However, from our analysis it appears that these optimal locations are not influenced by the use of rounded pricing. This does not exclude the possibility that some minor differences in the set of optimal location are possible. These may exist when, due to the rounding, the critical price of an important demand point is crossed resulting in a significant loss in demand. In the datasets presented here, no such instances appeared. Important to note is the fact that the relative importance of demand points depends to some extent on the level of aggregation. For a more detailed discussion on this subject see [12].

The drop-off mechanism that was presented above states that consumers generally do
not take into account the whole price of a product when making a purchase decision. Several explanations have been examined in literature, such as bounded memory capacity or left-to-right comparison. Applying this logic to the PrMAXCOV model by Plastria and Vanhaverbeke [10], it implies that the demand by consumers is in fact based on the whole part of the price and that they make abstraction of the decimals. This in turn means that the demand calculated by the model is an underestimation of the demand that will actually be covered.

Table 2 gives an indication of the impact on the solution of the PrMAXCOV, if the level-effect is considered. It shows that the revenue increases when the rationale of the drop-off mechanism is used. The \( p^* \), \( D^* \) and \( \Pi^* \) are the respective optimal price, demand and revenue that were generated using the original PrMAXCOV.

<table>
<thead>
<tr>
<th>Budget</th>
<th>Optimal price</th>
<th>Covered demand</th>
<th>Optimal Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p^* )</td>
<td>( p_c^* )</td>
<td>( D^* )</td>
</tr>
<tr>
<td>1</td>
<td>10.8194</td>
<td>10</td>
<td>10.99</td>
</tr>
<tr>
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<td>5.0279</td>
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<td>5.0577</td>
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<td>5.99</td>
</tr>
<tr>
<td>10</td>
<td>5.0554</td>
<td>5</td>
<td>5.99</td>
</tr>
</tbody>
</table>

Table 2: Comparison between revenue generated by the PrMAXCOV and the revenue generated using the level-effect, for the large case.

The difference between the original revenue (\( \Pi^* = D^* \cdot p^* \)) and the revenue incorporating the level-effect (\( \Pi^*_l = D_c^* \cdot p_c^* \)) can be attributed to two separate influences. On the one hand the price perceived by the consumers (\( p_c^* \)) is lower than the optimal price (\( p^* \)). This was determined using the method described above. This lower price leads to consumer demand (\( D_c^* \)) that is higher or equal to the demand covered in the optimal solution (\( D^* \)), following the third property of the full enumeration method [10]. Multiplying this rounded price and the associated demand it generates, gives the revenue based on rounded prices (\( \Pi^*_l = D_c^* \cdot p_c^* \)) which was discussed above and was shown to be lower or equal to the unrestricted optimal solution.

However, from the literature on the drop-off mechanism, it appears the consumers would not take into account the left most digits in prices when making their purchase decision. This provides an incentive for the retailer to maximize the digits that consumers do not notice (or
remember), as it will not affect the demand covered. In this case it means that the retailer will charge a price that is just below the next whole value \((p^*_r)\). The revenue incorporation of the level-effect \((\Pi^*_l)\) thus equals the price charged by the retailer \((p^*_r)\) multiplied with the demand \((D^*_c)\), based on the price that the consumers perceive \((p^*_c)\).

Noteworthy is the fact that the price that consumers perceive will not always be the rounded price as we defined it here. Consumers tend to make abstraction on the right most digits in the price, not just the decimals. If the product under consideration would be a fairly expensive product costing thousands or hundreds, it could be that even unit digits would be left out of the decision, as they then represent a relatively small part of the monetary cost. And the retailer would therefore have an incentive to maximize these unit digits. There have not been any studies examining this specific relation between price level and the measure of the drop-off, so we will assume here that only the decimals are not taken into account.

These results indicate that a firm could benefit from examining the specific (irrational) consumer behavior that its customers show. Understanding the way in which the firm’s customers process the price information could result in higher revenue for the firm or at least a better anticipation of the revenue that may be generated by charging a certain price.

3.2 Price ranges

In this section, we will turn to the second issue raised by Plastria and Vanhaverbeke [10] in their concluding remarks: “what would be the effect of price ranges on the solution of the model?” In this remark, they mention the possibility of the retailer, i.e. the firm, to be interested in certain ranges of price. Alternatively, a price range could be imposed by some governing agency.

The goal here will be to assess the impact that such price ranges may have on the revenue of firms. In the above sections, there was only one bound on the price, formulated in equation (10). A minimum price is defined, to assure that firms do not sell below cost. Up until now this \(p_{min}\) was held constant at 0. Implementing price regulations can be achieved quite easily in the model, by using equation (17) instead of equation (10):

\[
p_{max} \geq p \geq p_{min}
\]  

(17)

Implementing price ranges in the model requires only a minor alteration in the first step of the algorithm. Instead of considering all critical prices in the list \(P\), only the prices that fall within the permitted range will be considered. The sole consequence for the model will therefore be that the list \(P\) will become shorter. As no further changes are needed, all properties described by Plastria and Vanhaverbeke [10] remain valid. Note that in this section the original algorithm is used, and not the method proposed in the previous section using rounded prices. This is done to clearly show the isolated impact of the price regulations in
the solution of the model without interference from rounded prices.

Turning to the solution of the model, two cases can be distinguished. Firstly, in case the unregulated optimal price \( p^* \) falls within the permitted range, the regulated optimal revenue \( \Pi^*_r \) will be the same as the unregulated revenue \( \Pi^* \). The second possibility, where the optimal price falls outside of the authorized limits, requires some additional attention. As the optimal price is not feasible, there will be a loss in revenue for the firm.

Two lower bounds for revenue can be established, where it is presumed that the number of outlets \( B \) and the transportation costs \( t \) are held constant. Important to note is the fact that the competition is not restricted to the retailers price range. The \( p_c \) for all \( c \in C \) remain constant in what follows.

Assuming that \( p^* \notin [p_{min}, p_{max}] \) and that \( p^* \) is unique, we know that in any case \( \Pi^*_r \leq \Pi^* \). The two possible situations need to be examined separately. Firstly, we assume that \( p^* < p_{min} \). But \( p^* = \frac{\Pi}{CP(p)} \) and since \( p_{min} \in [p_{min}, p_{max}] \), we have \( CP(p_{min})p_{min} \leq \Pi^*_r \). It follows that

\[
\frac{CP(p_{min})}{CP(p^*)} \cdot \Pi^* < \Pi^*_r \leq \Pi^* \tag{18}
\]

Secondly, we assume that \( p^* > p_{max} \). Using the fourth property by Plastria and Vanhaverbeke [10] we can deduce that \( CP(p^*) \leq CP(p_{max}) \). Since \( p_{max} \in [p_{min}, p_{max}] \) we may write

\[
\frac{\Pi^*}{p^*} = \frac{CP(p^*)}{CP(p_{max})} \leq \frac{\Pi^*_r}{p_{max}} \tag{19}
\]

and we obtain

\[
\frac{p_{max}}{p^*} \cdot \Pi^* \leq \Pi^*_r < \Pi^* \tag{20}
\]

Equations (18) and (20) establish some lower bounds to the loss in revenue as a result of price regulations. In the case that the unregulated optimal price falls outside of the permitted range, a firm might choose one of the boundaries of the range. The question is whether this will actually be the best option in some situations. The firm may decide to charge any price that falls within the set boundaries. In that respect, some sensitivity analyses were performed, who’s results are featured in figures 3, 4, 6 and 7. Two illustrations of the lower bound are presented in figures 5 and 8.
All of the following results are based on the small case. The transportation costs were held constant at \( t = 0.01 \). Other sensitivity analyses were performed using different values for \((t)\), but the conclusions were the same. The numbers of outlets \((B)\) that were opened varied from 1 to 5.

Figure 3 shows the relation between the value for the upper bound of the price regulation \((p_{\text{max}})\), and the optimal price \((p^*_{\text{r}})\) that was determined by the model under the given regulation. The \(p_{\text{max}}\) varied between 56 down to 30 with increments of 2. The reader may notice that the scale on the horizontal, showing \(p_{\text{max}}\), is inverted. This to illustrate the relation between the optimal price solution and the restrictiveness of the upper bound on the price.

![Figure 3: Sensitivity of the optimal price \(p^*_{r}\) to varying \(p_{\text{max}}\).](image)

The left most part of the chart shows the unrestricted optimal prices. Without imposing a \(p_{\text{max}}\), the highest \(p^*\) is 52.41 \((B=5)\). Imposing a \(p_{\text{max}}\) of 56 will not have any impact in the solution of the model. The chart clearly shows that when the imposed range becomes restrictive \((p^* > p_{\text{max}})\), firms will charge prices that are equal to or slightly less than the \(p_{\text{max}}\). In this particular example, it appears that it is advantageous for the firm to charge less than what is allowed, which indicates that the difference in price is compensated by an increase in demand. However, restricted prices are relatively close to the upper bound. This shows that low prices will not result in sufficient increases in demand, although these might theoretically generate higher revenues. Whether or not the restricted prices remain equal to the \(p_{\text{max}}\) or fall slightly below depends to a large extent on the location of the demand points in the problem under consideration.

The impact on the firm’s revenue is shown in figure 4. We know from figure 3 that in
most cases $p^*_r$ is relatively close to $p_{\text{max}}$. Theoretically, a reduction in price could result in an increase of the covered demand, leaving revenue relatively unaltered. However, figure 4 shows that this is not the case in the example presented here. The firms are faced with declining revenue as the price regulation becomes more severe, meaning that $p_{\text{max}}$ becomes smaller. Again here the scale on the horizontal axis is inverted, to illustrate the loss in revenue in relation to the restrictiveness of the price range.

The curve for the case of $B=1$ remains unchanged for most values of $p_{\text{max}}$. This is because the optimal solution without a price range is 33.21. So the optimal revenue is only affected when $p_{\text{max}}$ is lower than this threshold.

![Figure 4: Sensitivity analysis of the optimal revenue $\Pi^*_r$ to varying $p_{\text{max}}$.](image)

With an increasing severity of the price restriction, it can occur that some outlets no longer generate additional revenue and thus become redundant. This can be seen in figure 4 when, at certain values of $p_{\text{max}}$, the curves for different values of $B$ start to coincide. Beginning at $p_{\text{max}} = 40$, the revenue generated by four and five outlets is the same. This means that the fifth outlet does not contribute to the retailer’s revenue. Even more striking is the fact that a fourth and fifth outlet are useless when $p_{\text{max}} \leq 34$. Under this very restrictive range, a firm can only open three demand covering outlets.

This remarkable effect can be explained by the fact that the prices charged by the competition are not subject to the restriction of the maximum price. Given the relatively low price charged by the new firm in the market, imposed by the restriction, most demand points will choose the entrant. The demand points that continue to prefer the competition will be those that are situated very close to them, benefiting from low transportation costs. In order
to compensate for higher transportation costs, the new firm would have to charge an even lower price. Given the restriction of uniform pricing, this would mean a decrease in revenue. Therefore, it is beneficial for the firm to ignore the demand points that are located very close to the competition and charge a higher price, rather than try to cover the entire market. An interesting avenue for further research could be to consider volatile prices by the competition.

Figure 5 illustrates how the optimal solution with price ranges compares to the lower bound that was established in equation (20). Obviously, the maximal revenue for the problem without the price ranges will always be larger or equal to the maximal revenue under price ranges. The lower bound on the revenue is only shown for the range in which the imposed maximum price is restrictive, when \( p^* > p_{\text{max}} \). Outside of this range the imposed price is non-restrictive, thus making lower bound without meaning because the unrestricted optimal revenue (\( \Pi^* \)) is feasible. The reader may notice that the maximal revenue with price ranges is the same as the curve for B=5 in figure 5. In this example, the revenue for a given maximum price is always higher than the calculated lower bound. The narrower the range (lower maximum price), the lower the maximal revenue will become for a given problem. This indicates that the reduction in price is not compensated by an increase in covered demand for this example. Other examined instances showed very similar evolutions.

Figure 5: Illustration of the lower bound for B=5 for different values of \( p_{\text{max}} \).

Until now, only the case of a maximal limit on the price was considered. Therefore, a sensitivity analysis was made on the variable \( p_{\text{min}} \). The results of this analysis are summarized in figures 6 and 7.

Depicting the relation between the variable \( p_{\text{min}} \) and the restricted optimal price (\( p^*_{r} \)), figure 6 shows that \( p^*_{r} \) reacts slightly differently when the price range is narrowed from below.
It appears that for this particular case, the optimal solution will less often revert to the $p_{\text{min}}$, when in the situation $p^* < p_{\text{min}}$. This can be observed by the fluctuation in the curves for each value of $B$. This means that the revenue is higher than the predicted lower bound in equation (18), which is shown in figure 8.

Figure 6: Sensitivity analysis on parameter $p_{\text{min}}$ for the value of the optimal price $p^*_r$.

The reader may notice that the curve for $B=1$ makes a considerable jump when $p_{\text{min}} = 46$. The optimal price increases considerably to 67, which is by coincidence the highest price of all competitors. This change in optimal price also coincides with a change in optimal location. Under the very restrictive minimum price the firm will locate at a location that is closer to some demand point than any competitor and charge the highest price in the market.

Looking at figure 7, it appears that price regulations imposing a minimum price of goods have a less negative impact on the firm’s revenue. The chart indicates that there exists a negative relation between $p_{\text{min}}$ and $\Pi^*_r$, but this is less severe than the impact by maximum price restrictions, shown in figure 4. This is the result of the fact that, for this example, the $p^*_r \neq p_{\text{min}}$ for most values of $B$.

Again, we can compare the optimal solution of the problem with the lower bound that was established in equation (18). The results are shown in figure 8 for the case of $B=1$. Notice that this latter curve is the same as the corresponding one in figure 7, but now compared to the unrestricted optimal solution and the lower bound. When comparing this with the analysis for the $p_{\text{max}}$, the optimal solution remains relatively close to the maximal revenue in the unrestricted case. This can be explained by the fact that loss in covered demand can relatively easily be compensated by a price increase.

This concludes our analysis of the effect of price ranges. As can be expected, when im-
Figure 7: Sensitivity analysis on the value of the optimal revenue $\Pi^*$ for different values of the $p_{\text{min}}$.

Figure 8: Illustration of the lower bound for B=1 for different values of $p_{\text{min}}$.

Posing additional restrictions on the solution of the problem, there will be a loss in optimality. It appears from the analyses presented here that imposing a minimum price on product will have less severe effects of the firm’s revenue than a maximum price.
3.3 Consumer segmentation

Up to this point, we have implicitly assumed that all consumers make their purchase decision in the same manner, using the same information in the process. However, we know from section 2.2.1 that this is generally not the case in the retail market. For example, Berné et al. [18] made the distinctions between high and low intensity price-information seekers. This means that some customers will go to great length to acquire price information, while others make little or no effort to get and use price information in their purchase decision. Therefore, this section is devoted to the differentiation between these consumer types.

In the PrMAXCOV, the consumer’s purchase decision process is described by the patronizing set \( N_i(p) \), for every consumer \( i \). A consumer will be covered by the new firm when it opens a new outlet \( j \in N_i(p) \), while charging the price \( p \). The patronizing set contains all facilities for which the critical price is at least the \( p \) that is being considered. The respective formulas for the patronizing sets and critical prices are given in equations (3) and (4). These can be determined for every demand point/customer \( i \) and every price \( p \in P \).

To facilitate the consumer segmentation, we will attribute to each demand point \( i \) a consumer type \( z_i \). In what follows we will assume that \( z_i \in \{1, 2, 3\} \), meaning that we assume the existence of three consumer types. If \( z_i = 1 \), then the patronizing set is defined, as in the original model, according to equation (4). These customer are fully informed and perfectly rational, and have the capacity and time to compare every possible outlet in terms of price and distance. Also, they are assumed to show novelty behavior.

The second customer type, if \( z_i = 2 \), is similar to type 1 with the exception that they present conservative behavior, rather than novelty behavior. As described by Plastria [14], these conservative customers will go on “patronizing the existing facility they patronized before” in case of a tie with the new facility. As mentioned in section 2, a tie can occur when, by accident, a customer’s total purchasing cost is equal for the existing and the new player. These customer’s patronizing sets are defined as follows:

\[
N_i(p) = \{ j \in J : p_{ij} > p \} \quad \forall i \in I
\]

In the framework of Berné et al. [18] these two customer types could be designated as high intensity price-information seekers. Where the two previous consumer types are quite similar, type 3 customers will determine their preferred outlet differently. The consumers with \( z_i = 3 \) can be considered as a non-informed or non-rational type. In the discussion of Berné et al. [18] they can be considered as the extreme case of low intensity price-information seekers. These are the customers that will simply shop at the outlet that is located closest to their location/home. This can be for a number of reasons. Perhaps the cost of the product is negligible compared to their total spending. They could also face restrictions in terms of transport modes available to them. Alternatively, they do not have access to price information.
or do have access, but not the time for intensive market research. A new firm’s outlet will cover the demand point if it is located closer than any of the competition’s existing locations. This is described in the following manner:

\[ N_i(p) = \{ j \in J : d_{ij} \leq \min_{c \in C} (d_{ic}) \} \quad \forall i \in I \]  

(22)

Many more types could be distinguished, based on any number of factors, such as age or profession. The three types described here only serve as an illustration of the concept of differentiation. For example, a maximal distance could be imposed for the customers of type 3, indicating that they will only consider the existing competition that is located nearby.

One important disadvantage to customer type differentiation in the PrMAXCOV is the fact that due to the alternative definitions for the patronizing sets, all properties defined by Plastria and Vanhaverbeke [10] become invalid. We can no longer rely on the relation between the critical prices and the covering sets, as the covering sets for demand points with \( z_i = 3 \) become independent of the price that is being charged. This implies that it is no longer guaranteed that the optimal price must be a critical price. Indeed, it might happen that no optimal price exists (see Plastria and Carrizosa [29]). Therefore, we shall use the rounded pricing method that was described above.

We will also use price ranges, as discussed in section 3.2, to avoid undesirable solutions to the problem. Without imposing an upper bound on the solution of the model, the optimal solution will always be an extremely high price. This is the result of the fact that the type 3 customers do not include the price in their purchasing decision. Without any limitation, the new entrant in the market will simply locate at the locations that are closer to type 3 demand points than any existing competing outlet and charge an exuberant price. This is not a desirable result, as even a lazy or indifferent customer will eventually react to extremely high prices. Therefore, we impose a maximum price on the solution of the model that is equal to twice the price of the most expensive competitor.

As we are obliged to use the full enumeration method the small case is used to illustrate the effect that consumer differentiation will have on the optimal solution of the problem. As there are 33 customers/demand points in the market, each consumer type is randomly attributed to one third of the customers, resulting in 11 customers for each type. As the maximal \( p_c = 65 \), we set \( p_{\text{max}} = 130 \) to avoid extreme solutions. To generate these solutions, we used the rounded pricing method that was described in section 3.1, with \( a = 0.01 \). The transportation costs were held constant at \( t = 0.01 \).

Looking at the results in table 3, it appears that the consumer differentiation has the largest impact when only one outlet is opened (\( B=1 \)). In that case, the optimal price with consumer differentiation is nearly double the solution where all consumers are assumed to be of type 1. In the other cases, prices are slightly higher or the same as in the original model. What is striking is the fact that there is a loss in revenue in all instances. This can
Table 3: Comparison between the optimal solution with and without consumer differentiation.

<table>
<thead>
<tr>
<th>Budget</th>
<th>Without differentiation</th>
<th>With differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>( p^* )</td>
<td>( \Pi^* )</td>
</tr>
<tr>
<td>1</td>
<td>33.21</td>
<td>20,457.36</td>
</tr>
<tr>
<td>2</td>
<td>39.79</td>
<td>30,001.66</td>
</tr>
<tr>
<td>3</td>
<td>40.65</td>
<td>32,235.45</td>
</tr>
<tr>
<td>4</td>
<td>50.26</td>
<td>33,925.50</td>
</tr>
<tr>
<td>5</td>
<td>52.41</td>
<td>36,739.41</td>
</tr>
</tbody>
</table>

be explained by the fact that some consumers of type 3 will automatically be attributed to the competition, which causes a decline in covered demand.

Looking at the graphical representation of the small case in figure 9 we can in part explain the increase in price when consumers are differentiated. For the case of B=1 the optimal location is site number 112, which can be seen at the center of the map. This is a centrally located outlet, relatively far from the competition and with many demand points in the vicinity. Remember that without differentiation all the demand points are of type 1. Therefore, given the optimal price the firm will reach a sizable portion of the total demand in the market.

However, with consumer differentiation the situation becomes very different. The optimal location will now move to site number 100, which is located at the north-west border of the market where some type 3 customers are located.

The effect of consumer differentiation is the clearest with B=1, but similar changes take place for other values of B. In general, the firm will rather locate close to the lazy or uninformed customers to whom it can charge relatively high prices. However, the demand that the firm gains with these customers, is lost for type 1 and 2 customers because of the higher price it will charge. Apparently in the example case presented here, the decrease in covered demand is not compensated by the increase in price, resulting in the loss of revenue that is illustrated in table 3.

In the right most column of table 3 the revenue (\( \Pi_{UD} \)) is calculated that would be generated by the firm when it uses the locations and price level that was calculated without consumer differentiation, in a market where consumer of all 3 types are present. In all instances the \( \Pi_{UD} \) is considerably lower than the maximal revenue with and without differentiation. This clearly illustrates the importance of consumer behavior knowledge for a new entrant in a competitive market. If the firm would predict demand and revenue based on the assumption that all consumer are perfectly rational and fully informed, while in fact a portion of the customers are subject to bounded rationality, then its actual results will be much lower than what the model would predict.

This exemplary case only serves as an illustration of the importance for a firm to un-
Figure 9: Graphical representation of the small case on a map of Brussels.

If a firm were to decide to enter a new competitive market under the assumption that all customers are fully informed and perfectly rational when this is in reality not the case, then the consequences on its results could be dramatic. As firms often use locational analysis models as a decision making tool, it becomes important that these models also include the differences in individual consumer decision making.

4 Conclusions and extensions

In most models within the field of operational research it is assumed that consumers are perfectly rational and fully informed. This is also the case for the maximal covering location problem with price decision developed by Plastria and Vanhaverbeke [10]. This rational behavior, however, is not what is being found by studies stemming from marketing research. Several studies indicate that most retail consumers do not have access to all the information required to make a fully informed purchasing decision. And in the event that they have the information it may still be that some consumers will not use it. In economics literature it is often called bounded rationality. If a firm opens outlets at locations determined by models assuming perfectly rational customers it could be that the actual coverage, and thus revenue, generated by the firms will be significantly different from that what was calculated by the model.
We started by comparing the consumer behavior as it was modeled in the PrMAXCOV with the behavior that appeared from marketing surveys on price knowledge and purchasing decision making. There appeared a considerable discrepancy between both approaches. Unlike what was assumed in the PrMAXCOV, a large portion of consumers does not make fully informed, rational decision. Often, they do not even have a good notion of the price that is being charged, let alone how this price compares to the competition.

With the aim of enhancing its applicability to real situations, we made three modifications to the original PrMAXCOV model. This served the dual goal of answering a question raised in the concluding remarks of the original paper as well as allowing an approach that is modeled more closely to consumer behavior as observed in real life.

With the first modification, we investigated the impact of working with rounded prices instead of the very precise solutions that were calculated by the original model. Adding this restriction of the model reduced the optimality of the solution, as can be expected. However, it appeared that the rounded solution can in some cases be different from the rounded value of the unrestricted optimal solution. Using the rounded prices allowed us to take the drop-off mechanism into account, where prices can be augmented to just below the next whole price, without loss of demand.

Secondly, the impact of price ranges was presented. In line with expectations, the revenue that could maximally be generated by the firm reduced drastically when limitations were placed on the prices that can be charged. A lower bound to the solution was determined and in the exemplary case considered here, the solution was consequently higher than this lower bound. The negative effect on the revenue, as a result of imposed price restrictions, is shown to be greater when an upper bound is imposed, compared with an obligated minimum price.

Finally, the model was adapted in order to allow for differentiation between several types of customers. For each type of customer a different decision rule was defined. This reflects the fact that in reality many consumers do not really take the price into account when making a purchase decision. Often they simply go to the store that is located nearest to them. The results showed that when customers with bounded rationality are present in the market, new firms will rather locate close to these demand points and charge relatively high prices, exploiting their indifference with regard to the price level.

One serious drawback of our approach is the fact that these modifications to the original model excluded the use of the intelligent enumeration method that was developed by Plastria and Vanhaverbeke [10]. Given the necessity of using the full enumeration method only smaller case were presented to illustrate the impact of our modification on the optimality of the problem’s solution. The development of new properties, taking into account the consumer based approach, would be an interesting avenue for further research.
References


