Evaluation by fuzzy rules of multicriteria valued preferences in

Agent-Based Modelling

P. L. Kunsch
Vrije Universiteit Brussel, MOSI Department, Pleinlaan 2 BE-1050 Brussels, Belgium
pkunsch@vub.ac.be

Ph. Fortemps
Faculté Polytechnique de Mons, Département de Mathématiques et de Recherche Opérationnelle, Rue de Houdain 9, BE-7000 Mons, Belgium
philippe.fortemps@fpms.ac.be

ABSTRACT
In multi-agent based modelling possible actions are evaluated on a current basis by multiple agents taking into account multiple criteria. Groups of agents with homogeneous preferences are considered. The dynamic decision taken by each group is handled in the paper by using fuzzy-control techniques. Fuzzy if-rules are used to combine valued pairwise preferences in the multi-criteria comparison of actions. The data to be provided for each group of agents in the model are the evaluation tables of the actions for all criteria, and the coefficients comprised in the [0, 1] interval representing the relative importance given to criteria. The merits of two fuzzy implications are compared for evaluating pairwise preferences. It is indicated in the paper how this methodology can be used in modelling uncertain dynamic environments.

Keywords: Agent-based modelling, pairwise multi-criteria preferences, fuzzy rules

1. SCOPE OF THE ANALYSIS
This paper provides a fuzzy-control methodology, developed by analogy with industrial practice described in Passino and Yurkovich (1998). As it is shown in the paper, this technique permits implementing in a ready-to-use manner multicriteria decision choices in multi-agent based modelling (ABM) (Ferber, 1999). Given a large number of agents with various preference profiles and a set a possible courses of actions, ABM provides a dynamic simulation tool. Its purpose is first to understand the emergence of collective behaviour patterns, and further to investigate the efficiency of external policies designed to influence these patterns towards some goals. Examples of such policies are: marketing
strategies (Wolfe, 2004) to making some first unknown product brands attractive to consumers; traffic-regulating policies to limit congestion in urban centers like described in De Smet et al. (1992); public policies to improve the quality of life in urban communities and influence moving patterns of populations like presented in Martens et al. (2004).

The initialising step in ABM consists in defining by means of clustering techniques groups of agents characterised by homogeneous properties, e.g. income, social status, etc.

In the following, we assume that this step has been completed. To describe the methodology it is sufficient to concentrate on one slice of time and one homogeneous group of agents. A sequence of decisions to be taken at successive single moments by many groups of agents with different properties can then be obtained by embedding elementary decisions into a larger dynamic model. Preferences and evaluations of actions will in general change by moving along the time axis, because of the existence of feedback loops as described in Sterman (2000). Therefore an action which looks attractive at a given moment in time, may become unattractive in the next one. For example, an initially quiet side-road of the highway may become very busy because too many people decide to drive on it.

For introducing the fuzzy-control methodology, it is sufficient to consider a frozen time and one agent, selected as a representative of the preference profile of the group to which he (she) belongs to. This representative gives a definite judgement on the absolute importance of each criterion used to measuring the merits and drawbacks of possible actions. Such a pseudo-static mono-agent framework, assumed for the analysis, can be defined as follows:

- **N** actions \( \left( a_1, a_2, \ldots, a_N \right) \) are evaluated with respect to **K** criteria, giving **K** raw evaluations, some are objective measurements (ex: in the congestion problem of De Smet et. al. (2002), the time in minutes spent in a traffic jam by the agent acting as a car commuter), some are subjective preference judgements (ex.: the preference for some wake-up time of the car commuter) \( g^k \), \( k = 1, K \) for each action. These evaluations can be normalised by introducing scores \( f^k(a_i) \in [0,1] \), \( k = 1, K; i = 1, N \) as follows:

\[
f^k(a_i) = \frac{g^k(a_i) - g^k_{\min}}{g^k_{\max} - g^k_{\min}}
\]

(1)

Where \( g^k_{\min} \), \( g^k_{\max} \) are the minimum, respective minimum, evaluations of criterion \( k \) for all \( N \) actions. Each score \( f^k(a_i) \) represents a valued preference score of action \( a_i \) with respect to criterion \( k \).
The representative of an homogeneous group of agents defines an absolute importance value $I_k$ for each criterion $k$, growing in the interval scale $[0,1]$ with increasing importance:

$$
\begin{align*}
I_k &= 0 & \text{if } k \text{ can be ignored} \\
I_k &= 1 & \text{if } k \text{ is essential}
\end{align*}
$$

(2)

We assume that all action evaluations and importance values of criteria are known without imprecision, i.e., that crisp numbers are available to represent them. These numbers can however be themselves be defuzzified values resulting from a previous fuzzy analysis. This is not however not essential in presenting the fuzzy-control methodology the authors have in mind. Fuzziness will be shown to creep in during the process of aggregating valued preferences for the set of criteria, so that global preference comparisons can be made.

In the following section 2 we briefly discuss why fuzzy multi-criteria decision-making has been chosen in this paper to aggregate valued preferences between pairs of actions. Section 3 defines the elements of fuzzy logic and fuzzy control used for the aggregation process. The basic reason is that fuzzy-rule systems are universal approximators for any arbitrary functions. Therefore they can be used in particular with preference functions, to represent preference relationships between pairs of actions in a multicriteria framework. In this approach the absolute weights, which correspond to semantic judgements about the importance of criteria, can be directly integrated in the aggregation of the results of rules. An example with two criteria is solved in section 4. In section 5 we describe further work on this topic and draw some conclusions.

2. FUZZY DECISION-MAKING

Many multicriteria approaches, the so-called outranking methods, are based on pairwise comparisons of preferences between pairs of actions, like explained in Roy (1985), and Vincke (1992). As early as 1970, Bellman and Zadeh (1970) have developed the basic techniques of fuzzy decision making. A review of fuzzy multicriteria decision making is given in Carlsson and Fuller (1996).

In the discrete case, common situations are that discrete alternatives need to be ranked, or that a best candidate alternative has to be eventually chosen. In ABM problems the scope is generally be different: no ranking of actions is needed, nor is the selection of a best action sought for. What emerges from the simulations is rather a probability distribution across different particular courses of actions, even if a single group of agents with homogeneous preference profiles is assumed. In De Smet et al. (op. cit.) a stochastic Markov matrix is set up, in which the rows and
columns correspond to different departures times: the elements of the matrix are derived from the pairwise comparisons of the valued preferences of different departure times from home to the workplace.

The problem is thus here to aggregate pairwise preferences across the set of actions, i.e., to calculate a pairwise preference value \(0 \leq P_{ij} \leq 1\) for any pair \((i, j)\) in the action set. Because the general approach is described here, the discussion can be limited to \(i=1, j=2\) with the purpose to discuss how to obtain \(P_{12}\).

A review of fuzzy operators used in aggregating preferences is given in Fodor and Roubens (1994), and Kacprzyk (1997). An interesting simple multicriteria technique for calculating pairwise preferences is given by the PROMETHEE method described in Brans and Vincke (1985), Brans and Mareschal (2002). This approach can be used as a starting point for ABM. Its main merit is its simplicity of use. The starting data are: on the one hand, the multicriteria evaluation table of \(m\) actions, and, on the other hand, a set of \(K>1\) criteria weights summing up to one. In addition, the decision-makers have to choose preference functions, also called pseudo-criteria (which is fact are related to normalised membership functions), which translate the differences between two evaluations in various units into a common scale \([0, 1]\) for all criteria. Different shapes are proposed for the preference functions, so that indifference thresholds and the progressive increase of the preference can be described with some details. There are however some difficulties in this approach, which have to do with the uncertainties in the choice of technical parameters: shapes, thresholds, and weights. A fuzzy version of PROMETHEE, able to cope with uncertainties in the evaluations of actions, and in the values of weights, by using fuzzy instead of crisp numbers, has been developed by Goumas and Lygerou (2000). Its purpose is however different from ours, as we still assume that preference data are crisp numbers. Though our purpose is different, we are brought to adopt the basic idea of PROMETHEE. The pairwise comparison of preferences, delivering a \(0 \leq P_{12} \leq 1\) value for each pair of actions, rests on the use of the knowledge of all differences \(d_{12}^k\) between the valued preferences of actions 1 and 2, for all criteria \(k=1,K\).

A difficulty in PROMETHEE can be easily remedied: when used in preference modelling, \(d_{12}^k\) must generally be rescaled in proportion of the average value of the evaluation of actions 1 and 2. For example, a difference of 1.000 EUR in the prices of two cars will be more important in terms of preferences, when the average price is 10.000 EUR, than in the case when it is 50.000 EUR. To avoid this bias, we propose to work with rescaled differences \(\tilde{d}_{12}^k\), calculated as follows:
\[
\tilde{d}_{12}^k = \left( \frac{(1 + w) f^k(a_1) + f^k(a_2)}{1 + w} \right) d_{12}^k
\]

where \( w \) is a weight factor, which can be chosen to be \( w \geq 1 \). When the half difference is small compared to the average value, one obtains from (3) \( \tilde{d}_{12}^k \approx \frac{d_{12}^k}{1 + w} \) is obtained from (3), and in the reverse case \( \tilde{d}_{12}^k \approx d_{12}^k \), when the difference becomes comparable to the average value of the two evaluations, as wished for.

In the following we abandon the notation \( \tilde{d}_{12}^k \), for indicating the rescaled difference \( d_{12}^k \) assuming that this operation has been performed as needed.

Another basic change from PROMETHEE is that we will use fuzzy operators indicated by the theory of fuzzy control, rather than a weighed sum.

3. ELEMENTS OF FUZZY LOGIC AND CONTROL

Fuzzy Logic (FL) is a mathematical technique to assist decisions on the basis of rather vague statements and logical implications between variables. FL is close to the natural language, this is why some people have called it ‘computation with words’. FL is very useful in many technical and economic applications in which imprecise and relatively vague judgments of experts have to be accounted for in a quantitative way as explained for business applications in Cox (1995).

Its basic ingredients are (1) ‘membership functions’ to represent ranges of possible values of a vague or imprecisely-known variable (‘fuzzy variable’ as opposed to ‘crisp variable’), and (2) ‘fuzzy rules’. The latter relate fuzzy variables, in the antecedent of the rule to draw some conclusions on the final results, in the consequent (or conclusion) of the rule.

(1) A ‘Membership function’ (m.f.) provides a possibility measure, called ‘membership grade’ (m.g.) for some affirmation. For example, the m.f. ‘MIDDLEAGED’ for a human being might be represented by a triangular m.f. as follows: the m.g. is 0 at 30 years (y), it peaks at 1 at 45 y, and it comes down to 0 at 60 y. This
triangular m.f. is represented by the triplet (30 y; 45 y; 60 y). In the same context other life-ages could be represented, e.g., ‘CHILD’, ‘YOUNG’, ‘OLD’. The interval of variation of the fuzzy variable is called the universe of discourse, in the given example for life-ages, it could be in the interval [0, 100] (years). The operation of translating imprecise variables into a fuzzy variable, represented by a m.f. is called ‘fuzzyfication’, e.g. using four m.f.’s describing different ages of life. Note that m.f.’s are different from probability distributions. For example, the total surface underneath any m.f. is not normalised to 1. They are defined in the framework of possibility distributions.

(2) A mapping between fuzzy variables is made possible by using ‘fuzzy rules’. In the given example, rules connecting the life-ages to the degrees of experience could be imagined:

(a) If AGE is ‘YOUNG’ then EXPERIENCE is ‘LIMITED’

(b) If AGE is ‘MIDDLE-AGED’ then EXPERIENCE is ‘APPRECIABLE’

etc.

In this 1-input, 1-output fuzzy system, the 4 life-ages (‘CHILD’, ‘YOUNG’, MIDDLE-AGED’, and ‘OLD’) would be represented by triangular m.f.’s and the experience levels by corresponding four trapezoidal m.f.’s (e.g., ‘VANISHING’, ‘LIMITED’, ‘APPRECIABLE’, ‘IMPORTANT’).

An implication operator defines the m.g.’s of consequents, as the result of the application of the logical fuzzy rules. The ‘min’ implication operator, corresponding to a logical ‘AND’ is commonly used in control systems of the Mamdani type (Fuzzy Logic Toolbox, 2001). For example, assuming that u is the m.g. of the input ‘YOUNG’ to the rule (a), and Y is the m.f. representing ‘LIMITED’ the ‘min’ implication operator will give as output for the rule a truncated trapezoidal m.f. of height min(u,Y).

Of course many other implications could be used. In this paper we will defend the opinion that the Kleene-Dienes implication represented by max (1-u, Y) can give good services in valuing pairwise preferences.

The consequents of all rules are aggregated using an additional aggregation operator, e.g. the ‘max’ operator corresponding to a logical ‘OR’. This operation will result in a composite m.f., which can then be ‘defuzzified’ to provide a single crisp value. Different defuzzification operators are used. The most common ones are: ‘centroid’
which comes to calculating the center-of-gravity of the m.f.; or ‘mom’ which correspond to the ‘maximum of modes’ in the resulting m.f..

4. THE PROPOSED FUZZY SYSTEM

We start from the valued pairwise preference of two actions for \( k=1, K \) criteria. In the graphics presented below, created and its fuzzy logic toolbox for use with MATLAB® (2001). We take \( K=2 \) for simplicity, and without loss of generality.

3.1. Fuzzification

Assume for illustration of the approach two actions \( a_1 \) and \( a_2 \), with raw evaluations \( g^k (a_1), g^k (a_2) \), for \( K \) criteria to be maximised on intervals \( [g^k_{\min}, g^k_{max}] \). Each interval represents the universe of discourse, in well-defined units, of each criterion \( k =1, K \). Our objective is to find the (positive) preference \( P_{12} \) of action \( a_1 \) over action \( a_2 \). For the following presentation with graphics we can assume in addition, without loss of generality and for the sole reason of simplicity, that the universe of discourse of each criteria can be rescaled to \([0, 1]\) intervals, by the linear transformation (1):

A difference of values for the pair of actions is computed as being:

\[
0 \leq d_{12}^k = \max(0, f^k (a_1) - f^k (a_2)) \leq 1
\]  

including the rescaling with (3). Of course, only positive preferences for action \( a_1 \) over action \( a_2 \) are needed.

We then introduce a fuzzy inference system with the following inputs and outputs:

- A first input for each \( k=1, K \) is given by the positive difference \( d_{12}^k \) calculated from (4). This first input can be measured over the universe of discourse \([0,1]\), according to the transformation (1), using four membership functions (m.f.’s): NULL, SMALL, MEDIUM, LARGE
- A second input for each \( k=1, K \) represents the ‘importance’ or ‘priority’ expressed by an absolute weight given to the \( k \)-th criterion, and measured in absolute terms on a \([0,1]\) scale (no normalisation is needed).
- The output is the preference level \( P_{12} \), measuring the non-negative preference of action \( a_1 \) over action \( a_2 \). For this output we can again consider four membership functions covering the universe of discourse \([0,1]\) of the
valued pairwise preference $P_{12}$: INDIFFERENCE, WEAK PREFERENCE, STRONG PREFERENCE, ABSOLUTE PREFERENCE.

For each $k=1, K$ the 2 inputs are connected to the common output by using four conditional rules with a logical ‘AND’ (‘min’ operator) in the antecedent part:

1. If $d_{12}^k$ is ‘NULL’ and WEIGHT is $WEIGHT(k)$’ then $P_{12}$ is ‘INDIFFERENCE’  \hspace{1cm} (5.1)

2. If $d_{12}^k$ is ‘SMALL’ and WEIGHT is $WEIGHT(k)$’ then $P_{12}$ is ‘WEAK PREFERENCE’  \hspace{1cm} (5.2)

3. If $d_{12}^k$ is ‘MEDIUM’ and WEIGHT is $WEIGHT(k)$’ then $P_{12}$ is ‘STRONG PREFERENCE’  \hspace{1cm} (5.3)

4. If $d_{12}^k$ is ‘LARGE’ and WEIGHT is $WEIGHT(k)$’ then $P_{12}$ is ‘ABSOLUTE PREFERENCE’  \hspace{1cm} (5.4)

In total there are thus $4*K$ rules for each pairwise preference of action $a_1$ over action $a_2$.

3.2 Fuzzy implication in rules

The next question is to choose implication operators as discussed in Fodor and Roubens (1994), in order to obtain the partial results of the $4*K$ rules. We will restrict our discussion to Mamdani-Sugeno (in brief Mandami implication, as we ignore the so-called Tagaki-Sugeno variant) and Kleene-Dienes implications, to be described in this section. The Mamdani implication is rather classical in control theory. It requests a logical ‘AND’, represented in the simplest way by the ‘min’ operator. More formally, indicating with index $i$ the $i$-th rule for the criterion $k$, calling $R_m$ this implication, $u_i$ the membership grade (m.g.) of the if-part, and $v_i$ the membership function (m.f.) on the conclusion side, the fuzzy output m.f. $\mu_i$ of the $i$-th rule is given by:

$$\mu_i = R_m(u_i, v_i) = \min(u_i, v_i) \hspace{1cm} (6)$$

$u_i$ is given by the logical ‘AND’ (‘min’) on the if-side of the rule in equations (3), i.e., calling $w_i$ the importance of the i-th criteria, and $g_i$ the m.g. of the first input to this rule:

$$u_i = \min(w_i, g_i) \hspace{1cm} (7)$$
In case action $a_2$ is preferred to action $a_1$ for this criterion $k$, the rules attached to $k$ will be inactive. The easiest way to eliminate them from the fuzzy system is to adopt the additional prescription:

$$\text{if } d_{12}^k < 0 \text{ then } w_i = 0 \quad (8)$$

This first approach does however not directly account for the importance and priority given to the criteria. It is why we propose to rather use the Kleene-Dienes implication, which is now described. We are brought to this proposition through the experience made in a previous paper Kunsch and Fortemps (2002). The authors have used there the Kleene-Dienes implication to aggregate the opinions of several experts with different credibility levels. The question was to evaluate the R&D budget to be spent for completing a project in radioactive waste management. In this example, calling $u_i$ the credibility of a given $i$-th expert used as an input, and $v_i$ the membership grade of the R&D budget announced by this expert, the fuzzy Kleene-Dienes implication $R_{KD}$ provides the fuzzy output m.f. $\mu_i$ of the $i$-th expert as a ‘max’ operator (corresponding to a logical ‘OR’), as follows:

$$\mu_i = R_{KD}(u_i, v_i) = \max(1 - u_i, v_i) \quad (9)$$

This choice is well adapted as, indeed, in case the credibility of the $i$-th expert would be very small, the m.f. would be flat with m.g.$=1$, indicating that no information can be gathered from him. In the aggregation step, of course, the ‘min’ operator will be used, so that the conclusion of this expert can be ignored. The same idea can be applied mutatis mutandis to our multiple-criteria case, by replacing ‘credibility’ by ‘importance’ according to (2), and keeping the previous definitions, the formula (9) can replace formula (6) in the preference evaluations.

### 3.3 Aggregation of partial rule-conclusions

Resulting from what has just been said, it is obvious that the operators for use in aggregating partial conclusions of rules will be a logical ‘OR’ (‘max’) in case of the $R_M$ implication, and a logical ‘AND’ (‘min’) in case of the $R_{KD}$ implication. As it can be noticed from results, the use of the $R_{KD}$ implication eliminates excessive sensitivities in the choice of weights which are always imprecise as shown in Kunsch and Fortemps (op. cit.), and also too many ‘hesitations’ in the decision-making process (as explained later).
3.4 Defuzzification and final result

The choice of the R\(\text{KD}\) implication (‘OR’) combined with the logical ‘AND’ for aggregating partial results of all rules, will normally give as global output a one-mode distribution. It is thus indicated to use the ‘maximum of mode’ (‘mom’) technique to obtain the final ‘crisp’ result. Note that on the contrary in the more traditional R\(\text{M}\) implication (‘AND’ with ‘OR’), the ‘center of gravity’ technique is more adapted.

Let us call \(m_{12}\) the maximum mode obtained in this way.

According to (8) not all criteria will be active in the comparison of action \(a_1\) over action \(a_2\). The relative importances of criteria active in the fuzzy system are to be compared to the total importance including all non-active criteria. The final valued preference of action \(a_1\) over action \(a_2\) is then given by:

\[
P_{12} = \frac{\sum_{\{d_i \geq 0\}} I_k}{\sum_{k=1}^{K} I_k}
\]

(10)

Of course to get the global picture in terms of preference, indifference, or incomparability between two actions, the preference \(P_{21}\), must be also be determined.

**Figure 1.** The four m.f.’s representing the first input (differences in values) to the rules attached to each criterion:

*NULL DIF, SMALL DIF, MEDIUM DIF, LARGE DIF*

5. **NUMERICAL EXAMPLE**
We now illustrate the methodology with a simple example with two criteria (K=2), so that there are 8 rules in total. The choice of m.f.’s is of course a delicate part of any fuzzy model, and it deserves much attention in practical problems. Neuro-fuzzy techniques are also available for calibration of m.f.’s. (Fuzzy Logic Toolbox, 2001).

In this example, we have found that trapezoidal m.f.’s were adapted to the first input and the output of each rule. They are represented in Fig. 1 and Fig. 2, respectively. Note that the output m.f. ought to be well overlapping, so that the lateral sides of the trapezes are not cutting each over, except at the level of m.g.=1, to avoid trivial (flat) outputs and the absence of a maximum mode.

![Figure 2. The four m.f.’s representing the common output of the rules for: INDIFFERENCE; WEAK PREFERENCE; STRONG PREFERENCE; ABSOLUTE PREFERENCE. Note the overlapping between the neighbouring m.f.’s.](image)

Fig. 3 shows the results of the analysis with the Kleene-Dienes implication \( R_{KD} \) for the values of the differences and of the two importance’s (weights) indicated by the cursors. It is assumed that both criteria are active in the comparison. Note that, if one of the rules is not ‘activated’ at all the first input is vanishing, and, whatever the weight, the output in the right screen is fully black. This means that this rule is ignored during the aggregation step using the ‘min’ operator. The same thing applies if the ‘weight’ of a criterion is brought to zero with the cursor.
Figure 3. *The results of the Kleene-Dienes implication* for two criteria and the values of differences and importance’s (weights) indicated by the cursors on the left hand side.

Figure 4. *A view on the preference surface (distance 1, distance 2) for criteria importance’s (0.1;0.8) using the Kleene-Dienes implication with ‘maximum of modes’ defuzzification.*

Figure 4 shows the preference surface in function of the two positive distances $d_{12}^1, d_{12}^2$. This figure illustrates the fact that fuzzy rules can reproduce any preference function, assuming the calibration effort has been done.

In comparison, figure 5 shows the preference surface obtained with the same choice of parameters with the Mamdani implication.
Figure 5. Comparison of the preference surface (distance 1, distance 2) for criteria importance’s (0.1;0.8) using the Mamdani implication with ‘center-of-gravity’ defuzzification.

The comparison confirms the observation that results with the Mamdani implication are less well-cut, and therefore preferences surfaces are smoother, than those obtained with the Kleene-Dienes implication. Smoothness may be a requirements in some problems, but we feel that in most ABM problems, this could imply two much hesitations between the two actions and therefore probability distributions of final behaviour patterns, which may become unrealistically large. Yet this allegation needs to be verified in each specific case, and we would not exclude the use of the one or the other inference, according to the problem under focus.

6. FURTHER WORK

The present methodology is of particular interest in the larger context of modelling in uncertain and changing environments. Variable preferences of multiple decision-makers often influence the course of possible actions within the system under scrutiny. The presented fuzzy-system can easily be embedded into a larger simulation model as explained in Fuzzy logic toolbox (2001).

This larger model can be implemented by considering extensions to the fuzzy system used as a driver for preferences and thus possible actions:
- **Upstream** by including an additional fuzzy stage for determining the importance of criteria from semantic statements of different groups of agents, or decision-makers, like: “This criterion is very important”, or “This criterion is rather marginal”. It is the very scope of fuzzy logic to be able to model such imprecise statements. This is also demonstrated in Kunsch and Fortemps (2002), where evaluations are made about the maturity level of technological projects from rather imprecise judgments of experts.

- **Downstream** by using the preferences to determine distributions of potential actions, by means of a fuzzy-control system with rules implementing the decision-making process.

Of course such development also requires a calibration of all parameters in the fuzzy system to mimic as good as possible the real behaviour of decision-makers.

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