Issue saliences and distance selection in spatial coalition formation models: an empirical investigation

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Abstract We examine the impact of three factors on the ability to predict political coalitions of spatial coalition formation models: (1) inclusion of issue saliences, (2) choice of distance function and (3) choice of solution method. Issue saliences quantify the relative importance that parties attribute to different policy dimensions. The second factor is the distance function that is used to calculate the difference between two parties’ positions. The classical application employs the most commonly used Euclidean distance in combination with the gravity center as consensus estimate. This is not the case in the consistent distance application, where the selection can be made between three function: Euclidean, squared Euclidean and rectangular. This choice also determines the consensus estimate. These first two factors are often neglected in existing coalition formation models. The three functions in the consistent distance application and the classical application, each either unweighted or weighted with issue saliences, gives us eight possible combinations to apply in four solution methods. These solution methods are different criteria by which to rank potential coalitions and subsequently determine the (set of) optimal coalition(s).

An empirical application including 28 democracies illustrates the impact these factors have on the predictive power of the spatial coalition formation models. The factor with the most important impact on the predictive power is the chosen solution method. Overall, the inclusion of issue saliences or the choice of distance function do not appear to have a significant influence on the predictive power. However, these factors do have significant interaction effects with the solution methods. For some methods, although not all, the issue saliences and distance function can have a significant influence on the model’s performance.

Keywords Coalition formation · issue saliences · distance function

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1 Introduction

Both private firms and political parties are often faced with the opportunity or even necessity to cooperate with other partners. When single political parties do not obtain a majority in a country’s legislative body, they need to form a coalition government. Private firms may need to work together with other firms to reduce overhead costs and increase their market share. This analysis will focus on coalition formation between political parties. Every time the need to cooperate arises, an agreement must be reached between the participating partners. This process has been studied with the help of a wide variety of techniques, often stemming from cooperative game theory (Hajdukova, 2006) or location analysis (de Vries, 1999).

A wide range of models has been developed through time, based on many different assumptions and with varying objectives. Early developments were solely based on the parties’ electoral power or size (Steunenberg, 1992). The main disadvantage to these power-oriented theories is that they completely exclude the parties’ policy preferences. The latter were the main focus of the family of policy-oriented theories, as they are sometime designated (de Vries, 1999). Common to most models within this family is the spatial representation of the policies. This means that policies are translated to a point in a multidimensional space.

Initially, models were limited to a single left-right policy dimension (Benoit and Laver, 2006; Warwick, 2006). This means that all policies are represented by a single point within the dimension’s range. Obviously, such an approach leads to loss of information when the parties in the coalition cooperate on more than one issue or policy. More recent work allows for any number of more precisely defined issue dimensions. How these are used in the formation process then depends on each particular model. In some cases the parties focus on the other parties in the coalition, while in other cases the emphasis lies on the location of the consensus that could potentially be reached.

An often made distinction in these one- or multidimensional models is made between complete and incomplete information models. Incomplete information models rely on the fact that parties involved in negotiations do not have access to all the factors they need, or the information is not available. The incomplete information and resulting uncertainty can, for instance, lead to bargaining delays (Diermeier and van Roozendaal, 1998; Martin and Vanberg, 2003; Golder, 2012). The present paper will concentrate on complete information models. We work under the premise that all the required information for the models is available, and that the included variables are the only information needed or used by the parties to reach an agreement. Within our framework, there is no uncertainty. We make this simplification to avoid interference of these uncertainties and the aspects that we are focusing on here.

Our aim is to examine the impact of some of the aspects involved in spatial representations. An implicit assumption made in most models is that all policy or issue dimensions are equally important, and that this is the case for all parties involved in the coalition. However, empirical results have shown that this is often not the case (Benoit and Laver, 2006; Warwick, 2006). For this reason, we introduce issue saliences as a measure of the policy priorities of individual parties. All solution methods are applied in at least two modes. In the first case, as is most common, distances are calculated under the assumption that all issue saliences are equally important to all parties. In the second case, issue saliences are allowed to vary between parties and/or issue dimensions. This raises the question whether or not the addition of issue salience as a variable has any effect on the ability to predict the correct coalition.

Another aspect that has not received much attention is the way in which distances are calculated in spatial models. The widely spread practice in spatial coalition modeling is simply to use the Euclidean distance. This is also the case in our classical application where Euclidean distance is used in combination with the gravity center as consensus estimate. We want to take a closer look at the potential impact of the choice in distance function on the predictive power of coalition formation models. Therefore, we apply a consistent distance application with three different functions: Euclidean, squared Euclidean or rectangular. Additionally, all three distance functions can be either unweighted or weighted with issue saliences. Therefore, the second main inquiry of this chapter is whether the choice of distance function affects the probability to predict the correct coalition.
For two of the models, minimal distance variance and maximal satisfaction, these are the only modifications that were made compared to their original formulation by previous authors. In the minimal aggregated distance and the maximin satisfaction models, we also made some alterations to other aspects. Overall, the main question(s) we ask here is which aspect has the largest, if any, impact on the ability of a model to predict the correct coalition: the solution method, the distance function, whether or not issue saliences are allowed to vary, or possibly a combination of two or more of these aspects.

A short overview of the historical development of coalition formation modeling is presented in section 2. Section 3 provides detailed information on the empirical case study and the solution methods. A statistical analysis is presented in section 3.3 as a way to answer our main research question. Concluding remarks are formulated in section 4.

2 Literature review

This literature review by no means presumes to comprise and discuss all efforts made in the field of spatial coalition formation theory. This would be a nearly impossible task and would also be quite redundant since several surveys of good quality already exist on the subject (e.g. Bandyopadhyay and Chatterjee (2006), De Vries (1999), Ray (2007) or earlier work by Straffin and Grofman (1984)). We simply want to show the reader the road that led up to the four solution methods that are used to answer our main research question. The solution methods themselves will be presented in closer detail in section 3.2. For the sake of clarity and uniformity, we have substituted the original authors’ notations with the symbols that are used in our models.

2.1 Early contributions in voting games

All the models used and mentioned in the following sections originate from game theory. The number of contributions on the subject has increased exponentially since the first of its applications in the social and economic sciences by Von Neumann and Morgenstern (1944) and Nash (1953), among others. Classically, a distinction is made between cooperative and non-cooperative game theoretic models. Our approach is in line with cooperative games where the players “are supposed to be able to discuss the situation and agree on a rational joint plan of action, an agreement that should be assumed to be enforceable” (Nash, 1953). We work under the premise that the negotiating parties make up a written coalition agreement at the end of the process wherein they define the nature and properties of their cooperation.

All models discussed are based on the principle of a characteristic function game where, for a set \( P = \{1, 2, \ldots, n_P\} \) of \( n_P \) players, the characteristic function \( v \) maps each subset \( A \) of \( P \), called a coalition, to a number \( v(A) \) (Kohlberg, 1971). In the general case “it is assumed that \( v(A) \geq 0 \) and that \( v(A) = 0 \) for all one-person coalitions, as well as for the empty coalition” (Kohlberg, 1971). A special class of characteristic function games is known as simple games or voting games, depending on the source. In these voting games “the worth of a coalition is restricted to \{0, 1\}” (de Vries, 1999), formally noted as \( v(A) \in \{0, 1\} \).

**Definition 1** (Boekhoorn et al, 2006) A simple game or voting game is an ordered pair

\[
G = (P, W)
\]

where

1. \( P \) is the set of players
2. \( W \) is a set of coalitions such that
   - \( W \neq \emptyset \) and \( \emptyset \notin W \)
   - If \( A \in W \) and \( T \supset A \) then \( T \in W \).
The set \( W \) comprises all \textit{winning} coalitions; defined as \( W = \{ A | v(A) = 1 \} \), as compared to the \textit{losing} coalitions that are not in \( W \) and belong to the set \( L \). The first axiom assures that the game is not trivial. The second axiom states that “any coalition that includes a winning coalition is itself winning” (de Vries, 1999), a property called monotonicity.

An important class within the family of voting or simple games are the so-called \textit{weighted voting games}. We assign to each player \( p \in P \) a weight \( w_p \geq 0 \), and introduce a threshold \( q \) defining the game as an \( n + 1 \) tuple (Boekhoorn et al, 2006).

\[
(q; w_1, w_2, \ldots, w_n, p)
\]

In a political context these weights \( w_p \) will often be the number of seats in parliament controlled by the party \( p \) or the percentage of the popular vote. A game is called \textit{symmetric} if all players in \( P \) have the same weight. The threshold \( q \), also known as quota (de Vries, 1999), defines the total required weight of the coalition for it to be winning. In a political environment this is most frequently interpreted as the number of votes or seats required for a (qualified) majority in parliament. This is noted formally as

\[
A \in W \iff \sum_{p \in A} w_p \geq q
\]

The initial three models presented in this section, and in all subsequent sections, fall within this class of weighted majority (voting) games. As a group, they are sometimes referred to as power-oriented (de Vries, 1999) or size-oriented (Steunenberg, 1992) coalition formation theories and mentioned in almost every article related to the subject. At the risk of rehashing established definitions, we discuss them quite extensively as their predictive performance will serve as the benchmark for our spatial models. An assumption common to all three is that “there is a positive payoff associated to the act of governing”, which “will be shared in some way among the members of a governing coalition” (Straffin and Grofman, 1984). This payoff will generally be the benefit of being part of the government and thus able to influence policy and/or controlling cabinet posts.

The first model was originally defined by Von Neumann and Morgenstern (1944) in their \textit{Theory of Games and Economic Behavior}. They found that, in the framework of simple games, “the value of a winning coalition does not increase when members are added to a winning coalition” (de Vries, 1999). When \( A \in W \) and thus \( v(A) = 1 \), we get for any \( p \notin A \) that \( v(A \cup \{p\}) \) is also equal to one. This implies that there exists an incentive to form coalitions where no member could be left out without losing the required majority to be winning. Such coalitions without any redundant parties are called \textit{minimal winning coalitions}. From this definition we can easily deduce that every subset of a minimal winning coalition is a losing coalition. The solution set of this theory consists of all minimal winning coalitions, denoted as \( W^{\text{Min}} \).

Based on the minimal winning coalitions, we can distinguish two special kinds of players. The first is the \textit{veto player} that “forms part of every winning coalition” (Boekhoorn et al, 2006), formally noted as \( p \in \bigcap_{A \in W^{\text{Min}}} A \). The opposite of the veto player is the \textit{dummy player} that is no member of any winning coalition, meaning that \( p \notin \bigcup_{A \in W^{\text{Min}}} A \).

Gamson (1962) and Riker (1962) almost simultaneously defined the more restrictive solution set of \textit{minimum size coalitions}, denoted as \( W^{\text{Size}} \). Formally, a coalition \( A \in W \) is of minimum size if and only if

\[
\sum_{p \in A} w_p \leq \sum_{p \in T} w_p \quad \forall T \in W
\]

This also implies that “a minimum size coalition is always minimal winning but this does not necessarily hold the other way around” (de Vries, 1999), which is noted as \( W^{\text{Size}} \subseteq W^{\text{Min}} \). In symmetric games all minimal winning coalitions are also minimum size. The philosophy of the solution set \( W^{\text{Size}} \) is based on the payoff’s associated with the participation in a coalition. The assumption made by Gamson and Riker...
Table 1 Seat distribution in the national parliament of Luxembourg

<table>
<thead>
<tr>
<th>Party (p)</th>
<th>Abbreviation</th>
<th>Number of seats (w_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christian Social People’ Party</td>
<td>CSV</td>
<td>24</td>
</tr>
<tr>
<td>Luxembourg’s Socialist Labour Party</td>
<td>LSAP</td>
<td>14</td>
</tr>
<tr>
<td>Democratic Party</td>
<td>DP</td>
<td>10</td>
</tr>
<tr>
<td>The Greens</td>
<td>GR</td>
<td>7</td>
</tr>
<tr>
<td>Action Committee for Democracy and Pensions’ Rights</td>
<td>ADR</td>
<td>5</td>
</tr>
</tbody>
</table>

is that “payoffs might be divided in proportion to the resources which each person (or player) brought to the coalition” (Straffin and Grofman, 1984). The resources provided by a political party to a governmental coalition are the votes in the legislative assembly. A rational player will then prefer a coalition in which the proportion of his own resources compared to the coalition’s total resources is maximized. As the payoff of forming a winning coalition is considered fixed in a given situation, this means that players will prefer a winning coalition with the least total resources. Translated to a political setting, this least resources principle means that “the coalition which forms should be the winning coalition with the smallest total number of votes” (Straffin and Grofman, 1984).

Leierson’s (1968) bargaining proposition, which is also known as the minimum number principle, is based on the idea that “as the number of actors increases there is a tendency for each actor to prefer to form a $A \in W_{\text{Min}}$ with as few members as possible”. His intuitive argument is that when there are fewer parties involved it becomes easier to conduct negotiations and hold coalitions together. The solution set, commonly noted as $W_{\text{Bar}}$ and known as the bargaining set, comprises all $A \in W_{\text{Min}}$ where $\forall T \in W_{\text{Min}}$ it is true that $|A| \leq |T|$. In this text $|A|$ denotes the cardinality of $A$, the number of parties in the coalition $A$. To illustrate the similarities and differences between these three power-oriented theories, we present example 1.

Example 1 This illustration is based on the seat distribution in the Chambre des Députés, the parliament of Luxembourg, that resulted from the national election of June 13th 2004 (Dumont and Poirier, 2005), as summarized in table 1. With the government only requiring a simple majority out of a total of 60 seats, we set the threshold $q = 31$.

Based on these values we can determine that, using the minimal winning coalitions model, the solution set would be the following: $W_{\text{Min}} = \{\{\text{CSV,LSAP}\},\{\text{CSV,DP}\},\{\text{CSV,GR}\},\{\text{LSAP,DP,GR}\}\}$. This is a relatively large prediction set given that there are only five parties in the game. It becomes smaller when using the minimum size criterion, which results in $W_{\text{Size}} = \{\{\text{CSV,GR}\},\{\text{LSAP,DP,GR}\}\}$. Both coalitions have a total weight exactly equal to the threshold. Looking at the set $W_{\text{Min}}$, we know that the minimal number of parties required to satisfy the threshold $q$ is two. This gives us the bargaining set $W_{\text{Bar}} = \{\{\text{CSV,LSAP}\},\{\text{CSV,DP}\},\{\text{CSV,GR}\}\}$.

These solution sets indicate that there is no veto player, but ADR is a dummy player as it is no member of any of the minimal winning coalitions. The coalition $\{\text{CSV,LSAP}\}$ that supported the formation of the Juncker-Asselborn I cabinet after the elections is an element of the minimal winning coalitions and the bargaining set, but is no minimum size coalition.

Similarly to example 1, we applied these three seminal models to all 28 countries included in our case study. For each of the 28 countries we used the results of the national election that took place during (or closest to) the period 2002-2004. This was the period during which the expert survey, that is used later on in this analysis, was operationalized. A detailed presentation can be found in section 3.1. The complete results are featured in table 2. The bottom line of the table indicates that from the 28 historical coalitions there were 22 minimal winning coalitions, 7 of minimum size and 11 in the bargaining set.

One of the main advantages of these models is their relative simplicity and the limited amount of required information. Only the set $P$, the weight $w_p$ for all $p \in P$ and the threshold $q$ are needed to determine
the solutions sets. This makes them very accessible and easy to use. A disadvantage of these models is that their solution sets often comprise many potential coalitions. The solution sets list the coalitions that comply with the various criteria, but fail to provide a clear (unique) prediction of the coalition that will be formed in reality.

Another disadvantage to the simplicity of these models is that the parties’ preferences in terms of policy are not taken into account at all. This can be regarded as a fundamental shortcoming, especially when modeling coalition formation in a political context. When political parties work together in a coalition government, the main objective will be to implement policy decisions in the areas over which the particular government or council has jurisdiction. It would be counterintuitive if parties choose to work together without any attention for the others’ positions on the relevant policies. As a response to this second criticism, the parties’ policy positions were introduced in the so-called spatial coalition formation models.

2.2 Spatial coalition formation models

The first clear use of the frequently used spatial metaphor as a shorthand for political and ideological beliefs was just after the French Revolution of 1789. The “physical positions of political groups in the National Assemblies, and later in the National Convention” (Hinich and Munger, 1997) became a proxy for their ideological beliefs. These allocations are relatively unchanged in today’s political spectrum, where parties on the left are generally more progressive, while conservative parties are usually located on the right.

Initially, this spatial representation of the parties was limited to a “one-dimensional left-right continuum based on their views about the extent to which government should intervene in the economy, redistributional issues, the desirability of legislating social behavior, and other issues” (Straffin and Grofman, 1984). Graphically, this means that the parties’ ideological positions are represented on a line by their ideal position $x^*_p$. This position is also known as the bliss point. This ideal position is then used by the parties to determine which other parties are located closest to them and thus preferred as coalition partner.

Although these one-dimensional models are important for the analytical understanding of policy preferences, empirical findings suggest that “a single ‘left-right’ dimension is not always enough to convey even the big picture” (Benoit and Laver, 2006). Benoit and Laver (2006) find that most theories “attempt to describe politics in positional terms using a limited number of underlying dimensions”. They emphasize on the limited number of dimensions “since adding ever finer-grained detail does not necessarily make for ever-more useful descriptions of the world” (Benoit and Laver, 2006). An extreme example of excessive specification would be to consider every line in the government budget as an issue dimension for political coalitions.

We define $I = \{1, \ldots, n_I\}$ as the set of $n_I$ issue dimensions “concerning the terms and conditions of the alliance that have to be settled by agreement” (De Ridder and Rusinowska, 2008). This means that we only consider the issues for which the coalition is authorized to make decisions by the participating parties. These can be different for every type of coalition, depending on the set goals or its jurisdiction in case of legislative or governmental coalitions. An answer to the question on the ideal number $n_I$ will depend on the area of application. In our case study, presented in section 3.1, we rely on existing empirical findings to determine $n_I$. If $n_I > 1$ the parties need to specify their ideal position $x^*_p$, which is defined by the coordinates $x^*_{ip} \in \mathbb{R}$ for each $i \in I$. Combining all this gives us the following definition of a spatial voting game:

**Definition 2** (Boekhoorn et al, 2006) A spatial voting game is an ordered triple

$$G = (P, W, \{x^*_p\}_{p \in P})$$

where

1. $(P, W)$ is a voting game
2. $x^*_{p_i} \in \mathbb{R}^{n_i}$ where $\mathbb{R}^{n_i}$ is an $n_i$-dimensional Euclidean vector space.

The main assumption in this spatial setting is that “a party would like to see its policies implemented” (Straffin and Grofman, 1984) and that the implementation of policies is only possible for winning coalitions. Unless a party gains a majority of votes it will have to work together with other parties which generally do not share all of its policy positions. To assure that the implemented policies are as similar to its own ideal policies, a party will choose “to join a coalition with other parties whose values and ideological positions are close to its own” (Straffin and Grofman, 1984).

Building on the work by Von Neumann and Morgenstern (1944), Axelrod (1970) defined the solution set of the connected minimal winning coalitions. A coalition is connected when it consists solely of parties with adjacent positions on the one-dimensional policy scale. This means that if parties $p_1$ and $p_3$ are member of a coalition $A$, and there exists a party $p_2$ that lies between them, then $p_2$ must also be a member of $A$ for it to be connected.

The main assumption made by Axelrod (1970) is that “the less conflict of interest there is in a coalition, the more likely the coalition will form” and the longer it will last. The assumption is that conflict originates from the heterogeneity in policies between the parties. When there exist differences between the ideal positions this will lead to conflicts as “two interacting players cannot simultaneously attain their most preferred policies” (Axelrod, 1970). Imposing that coalition members must have adjacent ideal policy positions reduces the possibility of conflict to a minimum.

Leierson’s (1968) minimal range theory is based on roughly the same rationale as Axelrod’s conflict of interest, namely that the differences between policy positions within the coalition should be reduced to a minimum. His aim is to minimize the largest disagreement between any two parties in the coalition, called the range. The result is that “a coalition will be formed, if all members agree that the range of this coalition is not larger than the range of any other winning coalition” (de Vries, 1999).

What binds these two models is the fact that they only use the ideal positions of the members in the future coalitions. No assumption is made with respect to the policies that will be enacted by the coalition once it is formed as a new entity. However, the policies implemented by a coalition can, and often will, be different from the constituent members’ ideal positions. This is the opposite case of the policy distance theory (de Swaan, 1973) where “coalitions are evaluated according to the proximity of the expected policy to the party’s program”.

A novelty in this model is the coalition’s expected policy position, which we note as $x^*_{A_i}$ and is based “on the policies and weights of the actors that are members of the coalition under consideration” (de Swaan, 1973). Most authors working on the policy distance theory will use the gravity center as a proxy for this expected policy point, although no explicit formula is provided in the original discussion. The model also implies that actors, or players in the game theoretic terminology, will prefer the coalition with the expected policy position closest to its own bliss point.

Increasing the number of policy dimensions ($n_i$) has the main benefit that the parties’ ideal position can be specified for each individual policy domain as the $x^*_{ip_i}$. But there are some disadvantages to the multidimensional framework. Simplifying properties of the one-dimensional setting no longer hold true, undermining solution concepts that are based on these properties. Straffin and Grofman (1984) provided an illustration of the difficulties that arise when Axelrod’s minimal connected winning solution is applied when $n_i > 1$. The multidimensional variant of the definition is “that a coalition is connected if it includes any party within the convex hull of its members” (Straffin and Grofman, 1984). Straffin and Grofman’s (1984) Norwegian example also shows that the solution set provided by this adapted definition can lead to an undesirable result in the sense that all minimal winning coalitions are connected. Although this is not a general principle, it eliminates the model’s initial benefit of reducing the often large solution sets of the power oriented models.

The opposite effect is found when the core solution (de Swaan, 1973) is extended to multiple dimensions. The core solution set comprises all undominated coalitions, those coalitions where none of the partic-
ipants would prefer or be better of in another coalition. Owen (1995) states that “a necessary and sufficient condition for a spatial game to have a non-empty core is that all median hyperplanes pass through the same point”. A median hyperplane is the multidimensional version of a two-dimensional median line, where the parties on each side of the line represent half of the total weight or number of seats in the game. He finds that if such a point exists, then it “is not only undominated, it actually dominates all other points” (1995). The core solution set thus comprises all winning coalitions with a non-empty core. This very stringent requirement is the reason why the core is often empty in a multidimensional setting, reducing its value as a solution method.

Schofield’s (1993; 1995) Heart solution can be seen as an attempt to define a core-like method in a multidimensional setting. He defines the compromise set as the convex hull of the ideal positions of the parties in a coalition. A core point then lies in the intersection of the compromise sets of all winning coalitions, if this intersection is non-empty. As this intersection is empty in most cases, the core will also be empty.

If such a core solution does not exist, Schofield (1993; 1995) turns to the so-called cycle set. This is the area bounded by the median planes (or lines). The term comes from the fact that cycles of preferences exist within it, as “for a point \( x \) in the cycle set there always exists another point in this set that dominates \( x \)” (de Vries, 1999). However, points outside the cycle set will always be dominated by at least one point from within the set. The heart is then defined as the union of the core sets and the cycle sets. We would add to this definition that only the core sets and cycle sets of winning coalitions would be considered, but Schofield does not impose this requirement.

As the reader might notice, the heart solution is a set of undominated (or nearly undominated) policy points, not a set of coalitions. This is because this model was originally not designed to predict coalitions. Another multidimensional extension of the policy distance theory, known as the maximal satisfaction solution (de Vries, 1999), was developed for the specific purpose of predicting coalitions. This solution method will be discussed in detail in section 3.2.3.

An example of a model that was developed from the start in a multidimensional setting is known as the winset model (Laver and Shepsle, 1990, 1996). Laver and Shepsle (1990; 1996) determine whether or not a coalition is in equilibrium based on the expected policy of the coalition. This is the same basic principle as the policy distance theory, but with a different definition of expected policy. Instead of taking the average of the players’ positions, Laver and Shepsle (1990; 1996) assume that when a party is attributed the cabinet portfolio that corresponds to a certain issue dimension it will implement its own ideal position or policy for that dimension. So the coalition’s policy will consist of a combination of player’s ideal policies in the individual dimensions. The coalition’s expected policy is then compared against those of all other possible alternative majority coalitions.

All the players in any alternative coalition will determine an indifference curve based on the policy of the coalition that is being examined. This latter policy is known as the status quo. This area within the respective indifference curves comprises all policies that are equally or more preferable than the status quo, for the given player. If there is no alternative majority coalition that has a policy which falls within the intersection of the preferable areas of its participants, then the status quo coalition is in equilibrium.

By no means did we include more than a fraction of the vast number of (spatial) coalition formation models in this short review. But we feel that most recent models can be traced back to those that are mentioned here, at least in some way.

2.3 Spatial model specifications

We have illustrated that since the seminal, and quite simplistic, power oriented theories there has been much effort towards more advanced modeling techniques in coalition formation. A fundamental change in approach has been the spatial interpretation. In turn, this entailed many more assumptions regarding the players’ behavior. Firstly, we examine the common practice of assuming that all issue dimensions are
equally important to all parties. Subsequently, we address the wide spread custom of using the Euclidean distance function.

### 2.3.1 Issue saliences

A very common assumption in spatial coalition formation models is that “all dimensions have equal salience, distances in all issues ‘count’ the same” (Hinich and Munger, 1997). Apart from a few recent exceptions (e.g. Debus (2008b; 2009)), most authors do not even mention the possibility that there could be a variation in the importance that the different players in the game attribute to different policy or issue dimensions. This supposition is actually made up of two distinct parts. The first part is the premise that all players in the game value each issue dimension the same. Political parties could measure the importance or salience of every issue dimension as the relative part of the government budget that is annually spent on that issue. For instance, if the defense budget is larger than the development aid budget, then the disagreements between parties on defense issues will be viewed as more important than differences on development aid. The second part of the supposition is that all issue dimensions are equally important. This is different from the first part since disagreements on different issues would be weighted equally and not, for example, by the relative part of the budget they represent.

The combined supposition that all issue dimensions are equally salient, and that this is the case for all players in the game, is contradicted by empirical findings. Benoit and Laver (2006) found empirical support for variations of issue saliences between parties, dimensions and countries. With regard to the latter, they found that taxes/spending and deregulation dimensions received the highest average importance scores in western European countries. In eastern Europe, these dimensions came in second place after the issue of joining the European Union². The environmental issue dimension was ranked lowest in both regions. Benoit and Laver (2006) also measured the salience that individual parties attribute to each issue dimension that was included in the survey. As a counterexample to the generally used supposition they provided the comparison of two parties from the Netherlands. The salience of the immigration dimension was almost twice a large for the *Lijst Pim Fortuyn* (LPF) as for the *Staatskundig Gereformeerde Partij* (SPF), while the exact opposite was true for the social policy dimension. Many other cases can be found in the results of the survey. Similar empirical evidence against the equality of issue saliences was presented by Warwick (2006).

A spatial coalition formation model that includes issue saliences is the generalization of the Winset model by Debus (2008b). His opinion that “one has to account for the programmatic positions and the issue saliences of political parties when analyzing patterns of government formation” (Debus, 2008b) is perfectly in line with our own. The indifference curves in his version of the Winset model have an elliptical shape, where the axes of the ellipses are always parallel to the axes of the issue dimensions. This complies with the second case, as defined by Hinich and Munger (1997), where “preferences are separable and issues have different salience”. Their first case is the special, but most frequently used, situation when preferences are separable and issues have equal salience.

In line with general convention, we employ the Euclidean distance function to illustrate how issue saliences are practically implemented in the remainder of this text. For now, we assume that preferences are made towards a given policy proposition \( \theta \in \mathbb{R}^n \). At this point, we assume that this \( \theta \) is fixed and formulated externally. The *disagreement* between party \( p \) and the proposal \( \theta \) in each policy dimension \( i \in I \) is defined as \( |x^*_{ip} - \theta_i| \). The *distance* between policy positions is an aggregate measure of the disagreements over all issue dimensions. When no specification is made which function is used to determine the distance, it is noted as \( d(x^*_{ip}, \theta) \). This notation is used in general properties that are not affected by the specific function that may be used. Two possibilities were found to incorporate issue saliences in a Euclidean-like distance between the ideal position of player \( p \) and the policy proposition \( \theta \). The first is the definition by Hinich and Munger (1997):
\[
\sqrt{\sum_{i \in I} s_{ip}(x_{ip}^* - \theta_i)^2}
\] (1)

where \(s_{ip}\) is the issue salience attributed to issue \(i\) by player \(p\). In this framework, issue saliences are defined as a percentage of importance that is attributed by each player and are normalized by

\[
\sum_{i \in I} s_{ip} = 1 \quad \forall p \in P
\] (2)

Other possibilities for the normalization of issue saliences are presented by Plastria and Blockmans (2014), such as the product-1 normalization. Allowing different issue salience normalizations would imply the inclusion of a fourth factor that could potentially influence the predictive power of the coalition formation models. We have not included this factor in the present text. The main reason being that the empirical estimation of the issue saliences is based on the sum-1 normalization (2). The second possible definition for a salience-weighted Euclidean-like (SE) distance function is presented by Debus (2008b) as:

\[
d_{SE}^p(x_{ip}^*, \theta) = \sqrt{\sum_{i \in I} (s_{ip}(x_{ip}^* - \theta_i))^2}
\] (3)

The only difference between functions (1) and (3) is that the \(s_{ip}\) is squared in (3) and that they are outside of the brackets in (1). In the remainder of this text, we will exclusively use (3) to calculated Euclidean-like distances. Because saliences \((s_{ip})\) should have a similar impact on the distance as the disagreement \((|x_{ip}^* - \theta_i|)\) in each dimension, they are both squared. The reader can also notice the subscript \(p\) in the notation of the distance. This addition compared to the regular Euclidean distance is necessary since the result of the function depends on the \(s_{ip}\) and thus on the player (or party) by which it is measured or perceived.

When a model uses the salience-unweighted Euclidean-like distance function, in line with Hinich and Munger’s (1997) first case, the \(s_{ip} = \frac{1}{n_I}\) for all \(i \in I\) and all \(p \in P\). This special case complies with the definition of an averaged Euclidean distance (D’Agostina and Dardanoni, 2009).

For the sake of completeness, we mention that Hinich and Munger (1997) also defined a third case where preferences are not separable. With separable preferences, the utility of a position in one dimension is not influenced by the player’s position in another issue dimension. Non-separable preferences would lead to elliptical indifference curves whose axes are not parallel to the dimension axes. The separability of preferences is one assumption that we continue to employ. One reason is that, as far as we know, there exist no large scale empirical estimations for saliences that allow to make a connection between the position in two or more dimensions. A second reason for excluding the case of non-separability is the computational difficulties with the calculation of the distances. One assumption that we continue to use in the present work is that of symmetric distances. A possible direction for future work could be to use asymmetric distances, where moves along issue dimensions in different directions can be valued differently by the parties.

2.3.2 Distance function

Our attention to this aspect in spatial coalition formation models was sparked by a somewhat remarkable statement. Boekhoorn et al. (2006) pose that although “there are neither theoretical nor empirical reasons to use the Euclidean norm”, it remains the standard norm. This last part of the statement is certainly true: the vast majority of spatial coalition formation models use the Euclidean distance function. From our review of the literature we also found the lack of a clear justification for this practice. They simply defined distance as being Euclidean, without any comment. However, because most authors might not have mentioned them, or been aware of them, does not mean that there do not exist any reasons for this preference towards Euclidean distances.
A potential explanation was found by Benoit and Laver (2006) who conducted an informal expert survey of researchers who are involved in spatial (political) modeling. An overwhelming majority answered that, in their opinion, Euclidean models are not grounded in some body of evidence about how real people think about politics. The mainly cited reason for using Euclidean distances is their computational tractability through differential calculus (Benoit and Laver, 2006).

The question could be raised in what contexts the Euclidean distance is the most appropriate distance function. D’Agostino and Dardanoni (2009) find that this is the case when the context requires a distance function that is both monotonically value-sensitive and monotonically order-sensitive. The former property means that the distance depends monotonically on the absolute difference between the values of their multidimensional coordinates. The latter is related to the order of the coordinates in a multidimensional setting. For example, suppose two parties in a four-dimensional issue space, with \( x_q^* = (x_{1q}, x_{2q}, x_{3q}, x_{4q}) \) and \( x_g^* = (x_{1g}, x_{2g}, x_{3g}, x_{4g}) \). Provided that the coordinates of \( x_q^* \) remain unaltered, but we can either swap coordinates \( x_{1g} \) and \( x_{2g} \) (case \( \alpha \)) or \( x_{3g} \) and \( x_{4g} \) (case \( \beta \)) with each other. A distance is monotonically order-sensitive when it grows monotonically with the distance between the swapped coordinates. The increase of the distance should be greater in case \( \alpha \) than in case \( \beta \) when \( |x_{1g} - x_{2g}| > |x_{3g} - x_{4g}| \) and visa versa.

In our setting, where issue saliences are introduced in the model, these properties are not required and often not satisfied. The main argument in favor of the Euclidean distance by D’Agostino and Dardanoni (2009) does not necessarily justify its frequent use in the class of models featured here. They clearly state that the property of monotonic order-sensitivity is “intuitively sound only in situations where different issues are regarded as equally important” (D’Agostina and Dardanoni, 2009).

An argument that justifies closer attention to the choice of distance function is made by Milyo (2000). Based on psychological findings, he concluded that individuals tend to look at differences in a multidimensional setting separately. This means that they look at each dimension separately to determine the difference between their own position and that of another party or person. There is little or no evidence of a cognitive step that would correspond to taking the square root of the sum of square differences, which happens in Euclidean distances. Milyo (2000) concludes that, when dealing with human perceptions, rectangular distances are better suited “for measuring distances on dimensions that cannot be separated analytically” (Benoit and Laver, 2006). The question that could be posed here is whether or not the psychological evidence from individual persons is also representative for the way that, for instance, political parties behave.

Given this limited and scattered evidence, we will distinguish between three distance functions. This will allow us to determine whether or not the choice has a significant impact on the predictive value of some spatial coalition formation models. Obviously, the first function is the Euclidean distance as defined in equation (3). The second function that will be used to apply the solution methods is the salience-weighted squared Euclidean-like (SS) distance:

\[
d_p^{SS}(x_p^*, \theta) = \sum_{i \in I} (s_{ip}(x_{ip}^* - \theta_i))^2
\]

(4)

It might seem strange to include both the ‘regular’ and the squared Euclidean distance, but this will be made clear in the next section. Based on Milyo’s (2000) reasoning, we have opted to include the rectangular distance which is also known as the city-block or Manhattan distance. The salience-weighted rectangular (SR) distance is defined as

\[
d_p^{SR}(x_p^*, \theta) = \sum_{i \in I} s_{ip}|x_{ip}^* - \theta_i|
\]

(5)

Similar to the Euclidean distance, the salience-unweighted squared Euclidean and rectangular distance are determined using the equations (4) and (5) with \( s_{ip} = \frac{1}{n_i} \) for all \( i \in I \) and all \( p \in P \).
2.3.3 The impact on theoretic consensus location

As mentioned in the literature review, many spatial coalition formation models assume that “the expected policy point of the coalition should be the weighted average policy point of the parties” (de Vries, 1999). To determine the difference between these expected policy points and the individual parties’ policy points, they use the salience-unweighted Euclidean distance. This combination will be designated here as the classical application. In the existing models, this classical application is salience-unweighted. To incorporate the remarks made in section 2.3.1, a salience-weighted version of the classical application is defined as the combination of the salience-weighted Euclidean distance with a salience-weighted gravity center:

\[
x_{iA}^G = \frac{\sum_{p \in A} w_p s_{ip} x_{ip}^*}{\sum_{p \in A} w_p s_{ip}} \quad \forall i \in I
\]

The consensus position \(x_{iA}^G \in \mathbb{R}^n_I\) for coalition \(A\) is defined by the coordinates \(x_{iA}^C \in \mathbb{R}\) for every \(i \in I\). The superscript \(G\) signifies that the estimation of the consensus was done by using the gravity center. The comparison between the salience-unweighted and salience-weighted classical application allows us to establish the impact of the inclusion of issue saliences in the four solution methods in section 3.2.

However, both versions of the classical application entail somewhat of an inconsistency. We know that center of gravity is “the point at which the sum of weighted squared distances to the given points is minimum” (Plastria, 2011). Nevertheless, the center of gravity is used in combination with Euclidean distances. This means that it is “the optimal solution to a quite different problem, because distances are very different from squared distances” (Plastria, 2011). Apart from this theoretical inconsistency, Debus (2008a) also finds that “the position of a (coalition) government is not necessarily located close to the center of gravity” of the parties’ ideal position. Based on earlier empirical evidence by Budge and Laver (1993) and Warwick (2001), this finding seriously undermines the classical application.

The relation between the definition of a salience-weighted distance function and the consistent estimation of consensus locations was the focus of our earlier work (Plastria and Blockmans, 2014). We will not repeat the details of that discussion, but it suffices to say that the consensus estimation depends to a large extent on the distance function that is used. This does not take away that there are some benefits to using the gravity center, which is presented in Plastria and Blockmans (2014).

Apart from the two versions of the classical application, we will use the consistent distance application. In this application, the choice of distance function determines the consensus estimation. We employ three methods described in our earlier work (Plastria and Blockmans, 2014) to estimate the coalition’s consensus position. The consensus location is estimated as the point that minimizes the sum of weighted distances between the ideal policy positions of the parties and the consensus position of the coalition. Of course, we will be consistent in the use of the distance functions, i.e. apply each function solely in combination with the corresponding consensus estimation. This gives us

\[
x_{iA}^C = \arg \min_{x \in \mathbb{R}^n_I} \sum_{p \in A} w_p d_p(x_{p}^*, x)
\]

where \(d_p(x_{p}^*, x)\) can be substituted with equation (3), (4) or (5), depending on the application. The superscript \(C\) signifies that the consensus estimation was done by using the minimization of distances, rather than the gravity center as in equation (6). The only case where \(x_{iA}^C\) can be determined by a closed form equation is for the squared Euclidean distance. The estimation of the coordinates for the consensus becomes
which resembles equation (6) quite closely. The difference between (8) and (6) is that in the former the issue saliences are squared while this is not true in the latter. The reader can easily see that the consensus estimation will be the same when using the consistent squared Euclidean distance application or the classical application when they are both salience-unweighted. In the remainder of the discussion, the notation \( x_{iA}^C \) will be used for the consensus position when the method of estimation can be either \( x_{iA}^G \) or \( x_{iA}^C \).

3 Empirical investigation

Based on the theoretical framework of spatial coalition formation models, we have identified several aspects which will be the focus of the empirical investigation. The initial step in the empirical investigation is choosing the appropriate source of the required data. The original data will then be converted into a form that is compatible with the definitions that are used in the models. Some selected empirical cases will initially serve to illustrate the application of four solution methods which can be used to predict a theoretically optimal coalition. This is followed by the analysis in which the impact of the various factors on the ability to predict the correct coalition is examined.

3.1 Empirical data for the case study

From the theoretical discussion in the previous section we know that it requires a large amount of information on the players. Some of this information is quite easy to gather, some is more difficult. The first task is establishing the players involved in a particular case, assembling the set \( P \). This is quite straightforward when modeling the formation of a government coalition. It suffices to take the list of players with representation in the relevant legislative assembly, either national, regional or supranational. In the case of government formation these players will generally be political parties, or can be individual members. When using the political parties as single entities, the underlying assumption is that all members of that party vote in line with the party directives. We will work under this assumption in our applications. Generally, such lists are readily available on government websites and published subsequent to an election by the European Journal of Political Research. The electoral weight \( w_p \) of each party, usually measured by the number of seats in parliament or percentage of votes in the election, can also be found in such sources.

Somewhat more difficult is establishing which policy or issue dimensions are relevant, defining the set \( I \). The question is which decisions will be made by the coalition. Those decisions will be the issue dimensions that fall under the responsibility of the coalition. This means that the participants in the coalition need to reach a consensus agreement, represented by the position, on those dimensions. In a political coalition the issue dimensions will be assumed to be the policy areas within its jurisdiction. This means that the issues will depend on the level of government. For national coalitions the dimensions will include very broad areas such as defense or economic policy. On the municipal level relevant issue dimensions can include garbage disposal, local culture or child care.

Defining the broad areas or responsibility of the (future) coalition allows the definition of the set \( I \). The next choice is the level of detail with which the set \( I \) is defined. The issue dimensions can be very aggregated, such as fiscal, social and environmental policies. This type of dimensions requires some form of estimation of the parties’ positions and saliences. Another possibility is to consider every line in the budget of the government as an issue dimension, since every amount needs to be agreed upon by the parties. The benefit here is that no estimation is required, the parties’ positions are represented by the amount they...
propose for each future expense. Although this latter approach is extremely detailed, it is often impossible to apply due to absence of the information and/or the time needed to acquire it. Also, some policies are difficult or impossible to quantify in terms expenditure, e.g. ethical questions regarding abortion or same-sex marriage. Our empirical application will rely on more general issue dimensions for which the parties’ ideal positions \( x_{ip}^* \) and issue saliences \( s_{ip} \) are estimated. The lists of dimensions for all individual countries used in the following case studies can be found in the survey results by Benoit and Laver (2006).

The available estimation methods that could supply the required information can roughly be divided in three categories: media coverage, party manifesto coding analysis and expert surveys (Helbling and Tresch, 2011). An example of the media coverage method is the National Political Change in a Globalizing World project (Kriesi et al, 2008). All articles that appeared in different press outlets relating to several elections and politics in general were analyzed. The texts were coded sentence by sentence with the help of a coding scheme based on fine-grained issues and later aggregated to broader issue dimensions (Helbling and Tresch, 2011). The best known example of the party manifesto method is the Comparative Manifestos Project by Klingemann et al (2006). This method is similar to the previous because it is also based on a coding scheme, which essentially translates text into quantifiable positions and saliences. The difference is the source of the text. Party manifestos originate directly from the party headquarters and are assumed to comprise the views of the party, often ratified by an assembly of the members. The media coverage is only indirectly linked with the parties since it has been written by journalists. There are several well-known examples of large scale expert surveys, including Benoit and Laver (2006), the Chapel Hill survey by Hooghe et al (2010) or Warwick (2006). The main difference with the other two methods is that, as the name suggests, experts in (domestic) politics are the source of the data, rather than coding texts. Most often, political scientists or observers are asked to locate parties on multiple issue dimensions. A comparison of the characteristics of each of these three methods was assembled by Helbling and Tresch (2011).

The question is now which of these three is most suitable in this context. A clear recommendation is made by Bäck and Dumont (2007), who find that expert surveys are better suited for use in coalition formation modeling than manifesto data. One objection against the use of manifestos is that parties might anticipate future coalitions. If this is the case, they might construct the manifesto so that it can be acceptable for the expected coalition partners. In spatial terms this means they could move their ideal positions towards the future partner(s), and do not reveal their real position. An alternative approach could be to move away from the expected future partner(s), so that the final consensus position after negotiations is closer to their original ideal point. Such strategies can interfere with the predictions of the coalition model. Obviously, this argument is also applicable for the media coverage method.

In light of the potential problems with the other two methods, expert surveys seem to be the best data source to test our hypotheses. To our knowledge there are only three surveys with a sufficiently large number of countries included to achieve a reliable empirical application, namely those listed above. The Chapel Hill survey (Hooghe et al, 2010) is not suitable for our analysis since it only provides estimates for the parties’ positions \( x_{ip}^* \) and not the issue saliences \( s_{ip} \). The remaining choice between Benoit and Laver (2006) and Warwick (2006) is simply based on quantity. Warwick (2006) covers fewer countries than Benoit and Laver (2006), making the latter the best option. The main survey will be supplemented by the case of Chile by Wiesehomeier and Benoit (2009), who used the exact same methodology as the original survey.

### 3.1.1 Data conversion for use in the model

One of the main reasons for choosing the empirical results of Benoit and Laver (2006) is the fact that they make the clear distinction between issue positions and issue saliences. As the influence of issue saliences is one of the main focuses of this text, this distinction in the survey is very important. Also the wide scope of 47 countries is a clear benefit. The vast majority of the experts who participated in their survey “were drawn from academia or research institutes and, with extremely few exceptions, excluded both journalists
and political actors” (Benoit and Laver, 2006). The issue dimensions included in the survey can be divided into two groups. The first group consists of the four substantive policy dimensions that were deployed in every country: economic, social and environmental policies as well as the decentralization of decision making (Benoit and Laver, 2006). The second group are policy dimensions that are related to the specific political context of the countries. For instance, in post-communist countries parties were evaluated on their treatment of former communists and the access granted to foreigners to purchase and own land. In countries that are EU member states, experts were questioned on the parties’ policy towards the scope of EU policy authority and their approach to EU governance. We refer to Benoit and Laver (2006) for a more detailed discussion on the indentification of expert and policy dimensions.

The ideal issue positions do not require any conversion, the values estimated by Benoit and Laver (2006) can simply be copied as input for the $x^*_i p$. Another important piece of information required by the model is the weight of each party ($w_p$). How these were chosen will be discussed for the individual countries as they largely depend on the timing of national elections and the institutional organization. We shall limit our analysis to the relevant parties ($p$) and issue dimensions ($i$) as identified by Benoit and Laver (2006). This leaves only the issue saliences ($s_{ip}$) as far as input for the model is concerned.

The policy importance measured by Benoit and Laver (2006) correspond to our absolute issue saliences ($s_{ip}^{abs}$), which have to be converted into the relative issue saliences ($s_{ip}$). This follows from the assumption that $\sum_{i \in I} s_{ip} = 1$ for each $p \in P$. Following Debus (2008b), “the relative weight of the policy dimensions for each party is measured by the absolute value of party $p$’s salience for dimension $i$ ($s_{ip}^{abs}$) divided by the sum of absolute saliences of all policy dimensions:”

$$s_{ip} = \frac{s_{ip}^{abs}}{\sum_{i \in I} s_{ip}^{abs}}$$

(9)

3.1.2 Selection of countries for the case study

In this section we provide a short introduction into the political and institutional organization for all countries that were included in the survey by Benoit and Laver (2006). If a country is not eligible for analysis it will be clearly motivated, the most common reason being a single party having a majority in parliament making the need to form a coalition irrelevant.

Apart from Chile, which is part of our case study, we shall not discuss in detail the other countries that were surveyed by Wiesehomeier and Benoit (2009). The reason why most countries surveyed by Wiesehomeier and Benoit (2009) are not mentioned here is that in many cases there is a single majority party. Chile was included in our case study because there was a clear majority coalition that formed the government.

The institutional organization is important to take into account when determining the electoral weight of the parties. In most cases we shall use the number of seats that each party occupies in the lower house of parliament, as the authority to ratify governments lies with this institution in most countries. If this general guideline is not used in the case for some particular country it shall be clearly mentioned and motivated. Another reason for not using vote shares, as provided by Benoit and Laver (2006), is the existence of electoral thresholds. This means that although a party was able to gain a percentage of votes in the election, it may be without representation in parliament when it did not reach the required threshold. For instance in Belgium parties must acquire at least 5% of votes in order to gain seats in parliament.

A part of the choice which weights will be used is the timing. The surveys by Benoit and Laver (2006) “were deployed between late 2002 and early 2004, with most occurring in 2003”. Therefore, we shall use the election result that falls within this period or the closest possible. This should provide the closest match between the electoral weight and the issue positions and saliences. We have consciously opted to use only one electoral result in combination with the empirical estimations for each country. Although this means
that our analysis is limited to 28 cases, we prefer this opposed to combining the empirical estimations with several electoral results so as to be able to predict consecutive coalition in each of the countries. This would augment the number of applications but, in our opinion, would also reduce the reliability of the analysis. Parties’ issue positions and saliences are not necessarily static and therefore not directly applicable in the coalition formation process in previous or subsequent periods.

In summary, the most common reasons for not including a country in our case study are listed below.

– Political instability and/or doubt about the electoral results during the relevant period (Albania, Belarus, Ukraine).
– Single party majority (Britain, Canada, France, Greece, Malta, Moldova, Russia, Spain, Turkey).
– A minority government without a fixed cooperation with the opposition (Romania).
– The government formation is compliant with a normal negotiation process between parties, but follows a different procedure (Bosnia and Herzegovina, Northern Ireland, Switzerland).
– Parties essential to the majority were not included in the survey (Croatia, Macedonia, Serbia).

Some countries were excluded for multiple reasons. Three countries (Denmark, New Zealand and Sweden) with a minority government were included in the case study because we could identify fixed parties that provided support. These were then considered as de facto part of the government. The overview of the countries included in our case studies can be found in table 2. We provide the year of the election that was used to determine the electoral weights, the name of the government, the historical coalition, whether or not this was a majority coalition, the privileged partner in case of a minority cabinet, number of parties with parliamentary representation \( (n_P) \) and the number of winning coalitions \( (|W|) \) for each of the 28 countries included in the case study. The historical coalition is the coalition which was actually formed and thus the one that should be predicted as the optimal solution in the various models. The six right most columns indicate whether or not the historical coalition was minimal winning \( (\in W^{\text{Min}}) \), minimal size \( (\in W^{\text{Size}}) \), and minimal number of parties \( (\in W^{\text{Bar}}) \). For each of these three power oriented theories the number of coalitions in the respective solution sets is given.
| Country          | Election year | Government name | Government coalition | \( | \) | Partners     | \( | \) | \( | \) | \( | \) | \( | \) | \( | \) |
|------------------|---------------|-----------------|----------------------|-----|-----------|-----|-----------|-----|-----------|-----|-----------|-----|-----------|-----|
| Australia        | 2001          | Howard III      | LPA, NP              | Yes | 3         | 4   | Yes       | 3   | No        | 1   | Yes       | 3   |
| Austria          | 2002          | Schussel II     | ÖVP, FPÖ             | Yes | 4         | 8   | Yes       | 3   | No        | 1   | Yes       | 3   |
| Belgium          | 2003          | Verhofstadt II  | VLD, MR, Sp.a, PS   | Yes | 10        | 502 | Yes       | 51  | No        | 20  | Yes       | 35  |
| Bulgaria         | 2001          | Saksburggotski I| DPS, NDS             | Yes | 4         | 7   | Yes       | 3   | Yes       | 1   | Yes       | 3   |
| Chile            | 2005          | Bachelet I      | PDC, PPD, PS, PRSD  | Yes | 6         | 29  | Yes       | 15  | No        | 2   | No        | 10  |
| Cyprus           | 2001          | Papadopoulos I  | DIKO, AKEL, EDEK    | Yes | 8         | 127 | No        | 7   | No        | 5   | No        | 2   |
| Czech Republic   | 2002          | Špidla I        | ČSSD, KDU-ČSL, US-DEU| Yes | 5         | 16  | Yes       | 5   | Yes       | 1   | No        | 2   |
| Denmark          | 2001          | Rasmussen I     | V                    | No  | KF, DF    | 8   | 128       | Yes | 16       | 3   | No        | 1   |
| Estonia          | 2003          | Parts I         | RE, RP, RL           | Yes | 6         | 32  | Yes       | 9   | No        | 2   | No        | 1   |
| Finland          | 2003          | Vanhanen I      | KESK, SDP, SFP      | Yes | 8         | 127 | No        | 11  | No        | 3   | No        | 1   |
| Germany          | 2002          | Schroeder II    | SDP, Green           | Yes | 5         | 16  | Yes       | 4   | Yes       | 1   | Yes       | 2   |
| Hungary          | 2002          | Medgyessy I     | MSZP, SZDSZ          | Yes | 4         | 8   | Yes       | 4   | Yes       | 1   | Yes       | 3   |
| Iceland          | 2003          | Oddson IV       | IP, PP               | Yes | 5         | 16  | Yes       | 3   | No        | 1   | Yes       | 3   |
| Ireland          | 2002          | Ahern II        | FF, PD               | Yes | 6         | 31  | Yes       | 5   | No        | 1   | Yes       | 5   |
| Israel           | 2003          | Sharon II       | Likud, Shinui, IHUD, NRP | Yes | 12        | 2023 | No        | 131 | No        | 50  | No        | 14  |
| Italy            | 2001          | Berlusconi II   | FI, AN, LN, UDC     | Yes | 10        | 512 | No        | 39  | No        | 6   | No        | 8   |
| Japan            | 2003          | Koizumi II      | LDP, Komei, NCP     | Yes | 6         | 28  | No        | 3   | No        | 1   | No        | 3   |
| Latvia           | 2002          | Repši I         | TB/LNNK, LPP, ZZS, JL| Yes | 6         | 32  | Yes       | 9   | No        | 1   | No        | 1   |
| Lithuania        | 2000          | Brazaukas I     | NUSL, LSDP           | Yes | 7         | 56  | Yes       | 5   | No        | 1   | Yes       | 2   |
| Luxembourg       | 2004          | Juncker-Asselborn I | CSV, LSAP          | Yes | 5         | 16  | Yes       | 4   | No        | 2   | Yes       | 3   |
| Netherlands      | 2003          | Balkenende II   | CDA, VVD, D66       | Yes | 9         | 254 | Yes       | 13  | Yes       | 2   | No        | 1   |
| New Zealand      | 2002          | Clark II        | Labour, PC          | No  | UF        | 7   | 63        | Yes | 6        | No  | 2         | No  |
| Norway           | 2005          | Stoltenberg II  | DNA, SPV            | Yes | 7         | 64  | Yes       | 14  | Yes       | 1   | No        | 1   |
| Poland           | 2005          | Marcinkiewicz I | PS, SRP, LPR        | Yes | 6         | 32  | Yes       | 9   | No        | 1   | No        | 1   |
| Portugal         | 2002          | Barroso I       | PSD, LDS            | Yes | 6         | 31  | Yes       | 5   | Yes       | 1   | Yes       | 2   |
| Slovakia         | 2002          | Dzurinda II     | SDKÚ, SMK, KDH, ANO | Yes | 7         | 63  | No        | 19  | No        | 2   | No        | 7   |
| Slovenia         | 2000          | Denovšek I      | LDS, ZLSD, SLS-SKD  | Yes | 8         | 126 | Yes       | 18  | No        | 8   | No        | 1   |
| Sweden           | 2002          | Persson III     | S                    | No  | V, Green  | 7   | 64        | Yes | 9        | No  | 1         | No  |

Table 2: Overview of the 28 countries included in the case study.
3.2 The four solution methods

The solution methods presented in this section are based on four different criteria by which to rank the elements of the set of winning coalitions. These four solution methods have the common property that they all rely on the estimation of the consensus positions of the potential coalitions. Other variations on these methods could be certainly thought of, either based on consensus estimation or not. The development of alternative methods can be an interesting direction for future research, but falls outside the scope of the present work. The reader should note that in all cases, which were used to illustrate the solution methods, the predictions confirm the coalition that was actually formed.

3.2.1 Minimal aggregated distance

One of the disadvantages of the power-oriented models mentioned in section 2.1 is that their solution sets can be quite large. Table 2 illustrates this, with Israel as the most extreme case. On itself this may not be a problem, but in many applications it might be preferable to have smaller solutions sets or even singletons. It is in this respect that the minimal aggregated distance solution was defined.

The fundamental principle underlying this method, and all methods here after, is that parties determine their preferences on the expected location of the consensus that will be implemented by the potential coalition. In section 2.3.3, we described two ways to estimate these consensus positions. In the classical application, the consensus position is calculated as the salience-weighted gravity center of the parties’ ideal positions. This is not the case in the consistent distance application, where the consensus position is chosen as the position that minimizes the sum of weighted distances between itself and the parties’ ideal position. The reader should note that weighted refers in this context to the electoral weights $w_p$. It is the latter definition that inspired this solution method.

The minimal aggregated distance method ranks all possible coalitions that have the required majority of the weights $w_p$ in the game. The criterion by which they are ranked is the aggregated distance calculated for each coalition $A$ as

$$\text{Agg}(A) = \sum_{p \in A} w_p d_p(x^*_p, x_A)$$

(10)

where $d_p(x^*_p, x_A)$ can be either the Euclidean, either squared or regular, or the rectangular distance. The distance, irrespective of the used function, is between the parties’ ideal positions $x^*_p$ and the estimated consensus position $x_A$. In our consistent distance applications, we use the same distance function to calculate this distance as the one that was employed to estimate the consensus position. This position is the result of the minimization in equation (7), which is the same function as (10). In the classical application we will always use equation (6) to determine $x_A$ in combination with the Euclidean distance.

The solution set is then defined as the coalitions that have the minimal aggregated distance, the lowest result for equation 10. The motivation behind this minimization is that smaller distances mean that parties have to make less concessions. The difference between their own ideal policy and the policy that will be enacted by the coalition is as little as possible. The distances are weighted by the $w_p$ to incorporate the fact that parties with, for instance, a large number of parliamentary seats contribute more to the majority of the coalition. So their concession should be accounted with higher importance than for smaller parties.

One benefit of this ranking method, rather than simply a yes or no solution like the power-oriented models, is the easy correction for external factors. Although many variables are included, there are many other aspects of coalition formation that have no influence in this model. Examples include personal relations between chairpersons of different parties, historical ties or electoral legislation. Take the case of Belgium where all democratic parties have agreed never to enter a coalition that included the nationalist Vlaams Belang (VB), a restriction called cordon sanitaire. It is perfectly possible that, without this information, the model predicts a coalition that contains the VB. If this is the case, one can simply apply a post-hoc
correction by scrapping the theoretical optimal solution with the first coalition in the ranking that does not contain the VB. This is in light of the combination of large-\(n\) methods (the solution method) and small-\(n\) method such as historical evaluation that is advocated by Bäck and Dumont (2007).

The solution set will generally be a singleton, although this is certainly not always the case. Depending on the case it is certainly possible that two or more coalitions have the same results for equation (10). A theoretical example is presented in theorem 1.

**Theorem 1** A non-minimal winning coalition can never be the only element of a minimal aggregated distance solution set.

**Proof**

Given a minimal winning coalition \(A\) and a party \(k \not\in A\), we know that the coalition \(A \cup \{k\} \not\in W_{Min}\). Both \(A\) and \(k\) are relevant, meaning that \(w_A = \sum_{p \in A} w_p \geq 0\) and \(w_k \geq 0\). Assume now that coalition \(A \in S\), with \(S\) being the minimal aggregated solution set. This implies that, given its consensus position \(x_A\), the result of equation \(\sum_{p \in A} w_p d_p(x_p^*, x_A)\) is minimal. This leaves two possible situations that may arise:

- \(x_k^* = x_A\): this guarantees that \(x_A = x_{A \cup \{k\}}\) and thus that \(d_k(x_k^*, x_{A \cup \{k\}}) = 0\). It is easily shown that

\[
\sum_{p \in A \cup \{k\}} w_p d_p(x_p^*, x_{A \cup \{k\}}) = \sum_{p \in A} w_p d_p(x_p^*, x_{A \cup \{k\}}) + w_k d_k(x_k^*, x_{A \cup \{k\}}) = \sum_{p \in A} w_p d_p(x_p^*, x_A)
\]

(11)

which means that both \(A\) and \(A \cup \{k\}\) are elements of \(S\).

- \(x_k^* \neq x_A\): this means that either \(x_k^* \neq x_{A \cup \{k\}}\) or \(x_A \neq x_{A \cup \{k\}}\) is true, or both. Whether we work under the case where \(d_k(x_k^*, x_{A \cup \{k\}}) > 0\) or the case where \(\sum_{p \in A} w_p d_p(x_p^*, x_A) < \sum_{p \in A} w_p d_p(x_p^*, x_{A \cup \{k\}})\) or a combination of both cases, we get that

\[
\sum_{p \in A \cup \{k\}} w_p d_p(x_p^*, x_{A \cup \{k\}}) = \sum_{p \in A} w_p d_p(x_p^*, x_{A \cup \{k\}}) + w_k d_k(x_k^*, x_{A \cup \{k\}}) > \sum_{p \in A} w_p d_p(x_p^*, x_A)
\]

(12)

which means that \(A \cup \{k\}\) can not be an element of \(S\).

If there are multiple solutions to (7), i.e. a non-unique estimation of \(x_A\), each one of them has to be compared to \(x_k^*\). Combining both possibilities leads us to conclude that a non-minimal winning coalition can only be an element of \(S\) when one of the redundant parties in the coalition has the same ideal position as the consensus that would be reached by the remaining parties in the coalition. The remaining coalition, without the redundant party \(k\), also has a majority of votes and because it has the same result for equation (10) it is also an element of \(S\).

Theorem 1 allows us to reduce the number of coalitions for which a consensus position needs to be estimated. Initially, it is only required to estimate the consensus positions \(x_A\) for all \(A \in W_{Min}\). The coalitions with the minimal results for equation (10) are then added to the set \(S\). This initial set can then be supplemented if there exist parties who have an ideal position that is equal to the consensus positions of the coalition(s) in the set. There is no necessity to estimate a consensus position for these non-minimal winning coalition since theorem 1 showed that this must be the same as for the corresponding minimal winning coalition.

Example 3 illustrates the minimal aggregated distance method using a historical case study. The ideal positions and issue saliences of the parties in this example, and all other examples in this text, were drawn from the expert survey by Benoit and Laver (2006) as discussed in section 3.1.
Table 3  Seat distribution in the national parliament of Austria.

<table>
<thead>
<tr>
<th>Party (p)</th>
<th>Abbreviation</th>
<th>Number of seats (w_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Democratic Party of Austria</td>
<td>SPÖ</td>
<td>69</td>
</tr>
<tr>
<td>Austrian People’s Party</td>
<td>ÖVP</td>
<td>79</td>
</tr>
<tr>
<td>Freedom Party of Austria</td>
<td>FPÖ</td>
<td>18</td>
</tr>
<tr>
<td>The Greens</td>
<td>Grüne</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 4 Illustration of the minimal aggregated distance solution for Austria, with the consistent salience-weighted squared Euclidean distance application.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Distance to consensus</th>
<th>Agg(A)</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPÖ</td>
<td>Grün 8,58 ÖVP 0,52</td>
<td>195,18</td>
<td>Yes</td>
</tr>
<tr>
<td>ÖVP</td>
<td>Grün 41,43 ÖVP 2,19</td>
<td>877,16</td>
<td>Yes</td>
</tr>
<tr>
<td>Grüne</td>
<td>ÖVP 6,82 ÖVP SPO 8,93</td>
<td>1154,68</td>
<td>Yes</td>
</tr>
<tr>
<td>FPÖ, ÖVP</td>
<td>Grüne 13,35 ÖVP 1,48</td>
<td>1170,77</td>
<td>Yes</td>
</tr>
<tr>
<td>FPÖ, Grüne</td>
<td>ÖVP 35,62 ÖVP SPO 5,58 10,97</td>
<td>1496,96</td>
<td>Yes</td>
</tr>
<tr>
<td>Grüne, ÖVP</td>
<td>FPÖ 16,63 ÖVP SPO 25,97 9,63 6,56</td>
<td>1655,09</td>
<td>Yes</td>
</tr>
<tr>
<td>FPÖ, Grüne, ÖVP</td>
<td>FPÖ, ÖVP, Grüne, ÖVP 20,99 30,72 7,81 8,18</td>
<td>2080,96</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Example 3  This illustration is based on the electoral situation in Austria after the federal elections of November 24th of 2002. As described by Fallend (2003), only 4 parties won seats in the Nationalrat. The electoral results are featured in table 3. In February 2003, the Schüssel II cabinet was formed and supported by the majority coalition {FPÖ, ÖVP} (Fallend, 2004).

Table 4 features the minimal aggregated distance solution method. The first column from the left lists all possible combinations of the four parties with representation in the federal parliament. The individual parties are also included to allow for the possibility that a single party wins an absolute majority and is able to form a government without coalition partners.

The model was applied using the consistent distance application with the salience-weighted squared Euclidean function. This means consensus locations were estimated using equation 8. The next panel in table 4 shows the distance between the consensus estimate and the ideal position for each party that participates in the given coalition. All possible configurations for the distance calculation mentioned in the previous sections can be applied in this method.

The aggregated distance \( \text{Agg}(A) \) in the third panel is defined as the weighted sum of the distances over all parties in the coalition. To avoid confusion, we repeat that the weighted sum signifies here that the distances are multiplied with the respective \( w_p \). This use of the term weighted does not refer to the distances themselves, in the second panel from the left, that are calculated using the salience-weighted squared Euclidean function. For instance, the \( \text{Agg}([\text{FPÖ}, \text{ÖVP}]) \) is equal to 195,18. This total is the sum of 154,43 attributed to the FPÖ and 40,75 to the ÖVP. In turn, the 154,43 by the FPÖ is the result of multiplying the distance (8,58) with the number of seats (18). For the ÖVP, 40,75 is the product of 0,52 and 79 parliamentary seats. The same method was applied for all other coalitions in \( \mathcal{W} \).

The right most column indicates whether or not the coalition is an element of the set \( \mathcal{W} \). The optimal coalition is then chosen from this set by looking for the coalition with the minimal aggregated distance. In this example, the lowest value is 195,18, corresponding to \{FPÖ, ÖVP\}. The prediction of the solution methods is correct and unique.
One might find that this solution method resembles the multidimensional Heart solution method (de Vries, 1999). It is certainly true that the main goal of reducing the number of possible solutions, compared to older models, is the same. They also share the methodology of defining a function that calculates a numerical value for each coalition that relies on the distances between the parties’ positions and the consensus location. However, there are some notable differences. Firstly, we have opted not to work under the hypothesis that “in a spatial coalitional game only coalitions in the Heart will be formed” (de Vries, 1999). The definition of the Heart solution can be found in section 2.2. This restriction was abandoned because empirical applications of the multidimensional Heart solution only resulted in 3 correct predictions out of 16 case studies (de Vries, 1999). Allowing all winning coalitions and not only those in the Heart should have a positive impact on the predictive power of the solution method.

The second difference lies in the calculation of the aggregated distance. De Vries (1999) defined this as the sum of distances, not the sum of weighted distances as we do here. Finally, multidimensional Heart solution was exclusively applied with the Euclidean distance and consensus location were always the gravity center of ideal party positions. The presented solution method, as were the three alternative methods that follow, was applied in eight different configurations. These eight configurations are the four ways to estimate the consensus positions, the classical application and consistent distance application with Euclidean, squared Euclidean or rectangular distance minimization, all either salience-unweighted or salience-weighted.

3.2.2 Minimal distance variance

This solution method corresponds to the first model established by Boekhoorn et al. (2006). They also used the gravity center as a proxy for the expected policy position, as is the case with many models. The difference with the previous ranking method exists in the fact that Boekhoorn et al. (2006) defined their conflict index “as the variance of the weighted Euclidean distances of all members of a coalition to the policy center of that coalition”. Again, the reader should note that weighted in this definition means that the \( w_p \) are included in the calculation of the gravity center, not issue saliences.

We will use the same criterion in this method, with the difference that we allow different distance functions, salience-weighted and -unweighted. This give the following criterion:

\[
\sigma^2(A) = \frac{\sum_{p \in A} \left( d_p(x_p^*, x_A) - \frac{\sum_{e \in E} d_e(x_e^*, x_A)}{|A|} \right)^2 |A|}{|A|} 
\]

Equation (13) is nothing more that the variance of the distances between the parties’ ideal positions and the estimated consensus location. The idea behind using the variance as a ranking criterion is one of equitable distribution of concessions. The distance between a party’s ideal position and the consensus position is the concession that it has to make, the ideological cost it has to pay to join the coalition. If certain parties have to make large concessions, move far away from their ideal position, while other parties move very little or not at all, frictions may develop within the coalition. Choosing the coalition with the smallest distance variance \( \sigma^2(A) \) minimizes the possibility that displeasure between partners occurs. In the optimal case, when \( \sigma^2(A) = 0 \), all parties in the coalition have to bridge the same distance. This means that they all pay the same ideological cost for joining the coalition.

Example 4 The case of Australia after the elections for the House of Representatives of November, 10\(^{th}\) 2001 illustrates this method in table 5. Only three national parties won seats in the parliament, namely the Labor Party (ALP), the Liberal Party (LPA) and the National Party (NP). The consensus estimation was made using the consistent salience-unweighted Euclidean distance application, which is also the function used to determine the \( d_p(x_p^*, x_A) \). The left part of the table is exactly the same as in table 4, listing all possible coalitions and the distances between ideal and consensus positions.
Based on the results we find that the coalition of the LPA and the NP has the lowest variance of all winning coalitions and is the optimal solution according to this criterion. This prediction was indeed the actual coalition that supported the *Howard III* cabinet (MacKerras, 2002).

### 3.2.3 Maximal satisfaction solution

The third method for determining the optimal coalition was originally developed as a multidimensional extension of the policy distance theory (de Vries, 1999). One of the main arguments for this extension is the fact that “simultaneously positioning a party or a coalition on two or more policy or ideological dimensions is more accurate than placing it on a one-dimensional scale since it contains more information” (de Vries, 1999). One of the main novelties of this model is the fact that “instead of striving for their highest individual gain, parties strive for maximal collective satisfaction” (de Vries, 1999). This is to avoid preference cycles that may occur when only individual rationality is used.

Similar to the original definitions of the previous two solution methods, the expected policy position of the coalition is effectively the weighted gravity center of the parties’ ideal policy positions and parties “use real Euclidean distances between the policy positions of the players and the expected position of the coalition” (de Vries, 1999). Again, we will generalize this by differentiating between the consistent distance application with three distance functions and the classical application. The clearest way to define this method is with the help of an example.

**Example 5** The illustration for this method is provided in table 6. It is based on the political situation in Hungary during 2003, following the national elections of April 21st 2002. This elections resulted in the formation of the *Medgyessy I* cabinet, supported by a coalition of the Hungarian Socialist Party (MSZP) and the Alliance of Free Democrats (SZDSZ) (Ilonszki and Kurtan, 2003). This left the Hungarian Democratic Forum (MDF) and the Hungarian Civic Party (Fidesz) in opposition as the only other parties with representation in the *Országgyűlés*, the federal parliament. The distances and consensus locations are calculated using the consistent salience-weighted rectangular distance application.

The first two columns are the same as with the minimal aggregated distance method presented in section 3.2.1. The first column lists all possible coalitions that can be formed with the parties in the national parliament. The second panel shows the distance between the consensus position and the ideal positions for all parties that take part in the coalition. Also similar to table 4, the last column indicates whether or not a coalition is winning.

The novel parts for this method are situated in the utility panel and the satisfaction score column. When calculating “the total satisfaction of a coalition, the first step is to ascribe utility to preferences” (de Vries, 1999). As mentioned above, it is assumed that parties will prefer coalitions with a consensus position that is closest to their own ideal policy. Based on this assumption, “distances will be normalized and inverted in order to ascribe utility to the coalitions” (de Vries, 1999). The normalization is based on two main reference distances.

---

**Table 5** Illustration of the minimal distance variance solution for Australia, with consistent salience-unweighted Euclidean distance application.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Distance to consensus</th>
<th>σ²(A)</th>
<th>W^'</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LPA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALP, LPA</td>
<td>2.62</td>
<td>1.71</td>
<td>Yes</td>
</tr>
<tr>
<td>ALP, NP</td>
<td>0.00</td>
<td>2.25</td>
<td>1.26</td>
</tr>
<tr>
<td>LPA, NP</td>
<td>0.00</td>
<td>1.34</td>
<td>0.45</td>
</tr>
<tr>
<td>ALP, LPA, NP</td>
<td>2.21</td>
<td>1.12</td>
<td>0.54</td>
</tr>
</tbody>
</table>
Table 6 Illustration of the maximal satisfaction solution for Hungary, with consistent salience-weighted rectangular distance application.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Distance to consensus</th>
<th>Utility</th>
<th>Sat(A)</th>
<th>VW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fidesz</td>
<td>MDF</td>
<td>MSZP</td>
<td>SZDSZ</td>
<td>Fidesz</td>
</tr>
<tr>
<td>Fidesz</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>MDF</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>MSZP</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>SZDSZ</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Fidesz, MDF</td>
<td>2.29</td>
<td>3.25</td>
<td>0.72</td>
<td>0.61</td>
</tr>
<tr>
<td>Fidesz, MSZP</td>
<td>5.49</td>
<td>0.00</td>
<td>0.34</td>
<td>1.00</td>
</tr>
<tr>
<td>Fidesz, SZDSZ</td>
<td>0.00</td>
<td>3.25</td>
<td>1.00</td>
<td>0.61</td>
</tr>
<tr>
<td>MDF, MSZP</td>
<td>1.80</td>
<td>1.00</td>
<td>3.76</td>
<td>0.78</td>
</tr>
<tr>
<td>MDF, SZDSZ</td>
<td>0.00</td>
<td>1.14</td>
<td>8.29</td>
<td>1.00</td>
</tr>
<tr>
<td>MSZP, SZDSZ</td>
<td>3.11</td>
<td>2.51</td>
<td>5.05</td>
<td>0.62</td>
</tr>
<tr>
<td>Fidesz, MDF, MSZP</td>
<td>5.49</td>
<td>2.51</td>
<td>5.05</td>
<td>0.62</td>
</tr>
<tr>
<td>Fidesz, MDF, SZDSZ</td>
<td>2.54</td>
<td>1.74</td>
<td>3.04</td>
<td>5.58</td>
</tr>
<tr>
<td>Fidesz, MDF, MSZP, SZDSZ</td>
<td>2.54</td>
<td>1.74</td>
<td>3.04</td>
<td>5.58</td>
</tr>
</tbody>
</table>

The first reference distance is the maximal distance between any ideal party position \(x^*_p\) and any estimated consensus position of a coalition \(x_A\) that is a possible solution, if \(A \in W\). This can be seen as a sort of a worst case scenario. It is defined as

\[
d_{max} = \text{Max} \{d_p(x^*_p, x_A) \mid p \in A, A \in W\}
\]

It should be noted that these distances are calculated using the same definition as in the estimation of the consensus position. The measurement of distance should be consistent throughout all steps of the procedure. In our example \(d_{max} = 8.29\), which is the distance between SZDSZ and the estimated consensus of coalition \{Fidesz, MDF, SZDSZ\}. The second reference distance is the minimal distance between any ideal party position \(x^*_p\) and any estimated consensus position of a coalition \(x_A^*\) that is a possible solution, if \(A \in W\). This can be seen as a sort of best case scenario.

\[
d_{min} = \text{Min} \{d_p(x^*_p, x_A) \mid p \in A, A \in W\}
\]

The \(d_{min}\) is equal to zero in our example, and occurs in three cases. This means that, for instance, the consensus location of the coalition \{MDF, MSZP\} is located at the ideal position of MSZP. The difference between these two references “can be seen as the range of the distances within the game” (de Vries, 1999). One of the assumptions made here is that parties will only gain utility if they participate in a coalition, so if \(p \notin A\), then \(u_1(A, p) = 0\). When parties are member of a coalition \((p \in A)\), the utility is defined as

\[
u_1(A, p) = 1 - \frac{d_p(x^*_p, x_A) - d_{min}}{d_{max} - d_{min}}
\]

We assume here that \(d_{max} \neq d_{min}\). One of the main advantages of this definition is that the interpretation is quite intuitive, as \(0 \leq u_1(A, p) \leq 1\). When the party is “farthest away from any of the winning coalitions” (de Vries, 1999), this means that \(d(x^*_p, x_A) = d_{max}\) and a utility equal to 0. The opposite situation exists when the given party and coalition are the closest combination possible, which results in the maximal utility of 1. When looking at the utilities in table 6, the reader may notice that there are several \(u_1(A, p)\) that are equal to 1. This can be explained by the use of the rectangular distance function in this illustration. One of the properties of a metric is the fact that when minimizing the sum of distance the solution will often be one of the destination points. This is the case here with three coalitions in which the MSZP takes part and one coalition with the Fidesz party. As these parties represent 46.1% and 42.5% respectively of the total...
number of seats in the Hungarian parliament they will have a large impact on any coalition in which they participate.

The optimal coalition will be the winning “coalition with the highest aggregated utility divided by the number of players in a coalition” (de Vries, 1999). The total utility is divided by the number of parties in the coalition to avoid the prediction of very large coalitions. If this would not be the case and “enough players are added to a coalition, the satisfaction will increase”, “even though the policy distances and weights are incorporated in the computation of utility” (de Vries, 1999). The satisfaction score of a coalition is computed as

$$Sat_1(A) = \frac{\sum_{p \in A} u_1(A,p)}{|A|}$$

These satisfaction scores are shown in the second column from the right in table 6. The coalition with the maximal satisfaction score is \{MSZP, SZDSZ\}, which was the coalition that was formed in reality. The $Sat_1$-score of 0.80 is calculated as the average of the utility of MSZP that is 1 and the utility of SZDSZ which is 0.61. In this instance, the prediction of the model is correct. The reader might wonder why the $Sat_1$-score is calculated as a regular average. An alternative could be a weighted average, where the $u_1(A,p)$ are weighted by the $w_p$. One reason against employing such a weighted average is that it benefits large parties disproportionately. Parties with relative large $w_p$ already have a relatively big impact on the estimation of the consensus, either through the classical application or by distance minimization, and rightly so. Parties with many parliamentary seats often dominate negotiations and this should be built into the model. But giving them again a larger impact in the $Sat_1$-score would almost remove any impact by the smaller parties and this is not desirable. Smaller parties can have a notable impact on coalition formation, especially when they provide the few, but necessary, remaining seats that are needed for a majority.

### 3.2.4 Maximin satisfaction solution

In this section we present an adaptation of the maximal satisfaction that was presented in the previous section. This method is more oriented towards the preferences of the individual parties and less towards the collective utility. One difference exists in the normalization of the distances between the consensus and ideal positions, which results in an adjusted definition of the utilities. Another adjustment was made in the definition of the satisfaction score. Let us again explain this method by an example.

**Example 6** This illustration is based on the political situation in Bulgaria after the elections of July 2001 (Spirova, 2010). This election resulted in the first government of Simeon Saksokoburggotski, supported by a coalition of the National Movement for Simeon II (NDS) and the Movement for Rights and Freedom (DPS). The NDS won exactly half of the parliamentary seats in the election, but needed to cooperate with the DPS as they did not have an absolute majority. This left the Bulgarian Socialist Party (BSP) and the United Democratic Forces (ODS), as the only two remaining parties with parliamentary seats, in opposition. The estimations were made using consistent salience-weighted squared Euclidean distances application.

The division of table 7 is very similar to table 6 which illustrated the maximal satisfaction method. Corresponding, the first column lists all possible coalitions. The left panel shows the distances between the consensus position and the ideal positions of the parties that participate in the coalitions, limited to the coalitions that have an absolute majority in parliament. Whether or not a coalition is winning can be seen in the rightmost column.

The first difference between this method and the previous one lies in the calculation of the utilities. Correspondingly, the first column lists all possible coalitions. The left panel shows the distances between the consensus position and the ideal positions of the parties that participate in the coalitions, limited to the coalitions that have an absolute majority in parliament. Whether or not a coalition is winning can be seen in the rightmost column.

The first difference between this method and the previous one lies in the calculation of the utilities. Similarly, the utilities are calculated by normalizing the distances. Unlike the collective approaches in the previous method, where the maximal and minimal distances are chosen from all possible combinations of
Table 7 Illustration of the maximin satisfaction solution for Bulgaria, with consistent salience-weighted squared Euclidean distance application.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Distance to consensus</th>
<th>Utility</th>
<th>Sat2(A)</th>
<th>YW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BSP, DPS, NDS, ODS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSP</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DPS</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDS</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ODS</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSP, DPS</td>
<td>8.31, 1.30</td>
<td>0.95, 0.13</td>
<td>0.13</td>
<td>Yes</td>
</tr>
<tr>
<td>BSP, NDS</td>
<td>4.49, 0.17</td>
<td>0.73, 1.00</td>
<td>0.73</td>
<td>Yes</td>
</tr>
<tr>
<td>BSP, ODS</td>
<td>8.08, 3.16, 1.48</td>
<td>1.00, 1.00</td>
<td>0.00</td>
<td>Yes</td>
</tr>
<tr>
<td>NDS, ODS</td>
<td>1.15, 6.00</td>
<td>0.25, 1.00</td>
<td>0.25</td>
<td>Yes</td>
</tr>
<tr>
<td>BSP, DPS, NDS</td>
<td>12.90, 0.81, 10.26</td>
<td>0.00, 0.51, 0.19</td>
<td>0.00</td>
<td>Yes</td>
</tr>
<tr>
<td>BSP, DPS, ODS</td>
<td>8.02, 0.75, 7.44</td>
<td>0.00, 0.56, 0.73</td>
<td>0.00</td>
<td>Yes</td>
</tr>
<tr>
<td>BSP, NDS, ODS</td>
<td>12.08, 5.30, 0.81</td>
<td>0.17, 0.56, 0.51</td>
<td>0.00</td>
<td>Yes</td>
</tr>
</tbody>
</table>

winning coalitions and the parties that participate in these coalitions, a more party oriented approach is used here. We define the set \( A_p \) that comprises all coalitions \( A \in W \) of which party \( p \) is a member. Using this set \( A_p \), we can present our alternative definition of the minimal and maximal distances as

\[
d_{\text{min}}(p) = \min \{ d_p(x_p^*, x_A) \mid \forall A \in A_p \} \quad \forall p \in P
\]  

and

\[
d_{\text{max}}(p) = \max \{ d_p(x_p^*, x_A) \mid \forall A \in A_p \} \quad \forall p \in P
\]

These alternative definitions imply that parties will only take the coalitions into account in which they themselves take part and choose the worst and best case from this set. These two reference numbers are then used in the same manner as in the previous section to calculate the utility of a party towards a coalition:

\[
u_2(A, p) = 1 - \frac{d_p(x_p^*, x_A) - d_{\text{min}}(p)}{d_{\text{max}}(p) - d_{\text{min}}(p)}
\]

Again, we assume here that \( d_{\text{max}}(p) \neq d_{\text{min}}(p) \). Also similar to the previous method, we know that \( 0 \leq u_2(A, p) \leq 1 \). The \( u_2(A, p) = 0 \) either when \( p \notin A \) or when the coalition \( A \) has the consensus position which is located furthest away out of all coalitions in the set \( A_p \). Conversely, \( u_2(A, p) = 1 \) when the consensus position is the closest to the party’s ideal position out of all coalition in \( A_p \). The reader may notice that in each column of the utility panel in table 7 there is at least one \( u_2(A, p) = 0 \) and one \( u_p(A) = 1 \). This is because parties base the utility of the coalitions solely on their own preferences and not on the whole of the preferences of all the parties towards all of the winning coalitions.

The second main difference with the previous method lies in the definition of the satisfaction score of the coalitions. In section 3.2.3, the \( \text{Sat}_1(A) \) were determined by dividing the aggregated utility of each coalition by the number of parties it comprises. This is based on the assumption that parties want to maximize the total utility, while avoiding that the coalition becomes too large and difficult to manage. Here we introduce the assumption that parties want to maximize the utility of the party in the coalition that is worst off. This means that we define the satisfaction score as

\[
\text{Sat}_2(A) = \min_{p \in A} u_2(A, p)
\]
The optimal solution is then chosen as the coalition with the highest satisfaction score, i.e. the coalition for which utility of the least off party is maximized. This is why we call it the maximin satisfaction method, as we want to maximize the minimal utility of all parties in the coalition. Returning to our illustration in table 7, the optimal solution is \{DPS, NDS\} which was the coalition that was formed in reality. In this case, the prediction of the model is correct.

3.3 Statistical analysis

Our main research goal is to establish which, if not all, of the three factors (solution method, distance function and issue saliences) has an impact on the probability to predict the correct coalition. But before we can do this, we need to establish the overall predictive power of the solution methods in their various configurations. In order to be able to do this, we need a benchmark to compare against. The power oriented models serve as this benchmark, as mentioned in section 2.1.

3.3.1 Comparison against the power oriented models

The reason for choosing the power oriented models as the benchmark for predictive power is that they require very little data, especially when compared to the variables required to apply the methods presented in section 3.2. The only information required to apply the power oriented models is the set of parties \(P\), each party’s electoral weight \(w_p\) and the required majority threshold \(q\). One of the pitfalls in confronting such different types of models with each other is choosing an appropriate criterion. Using obvious criteria such as the number of correct predictions or the success rate will often result in misleading conclusions (Steunenberg, 1992). The success rate, as the number of correct predictions divided by the total number of cases or countries, does not take the size of the solution set into account. The highest success rate is achieved by simply predicting all possible coalitions, which adds no value to the understanding of coalition formation at all.

The statistical test by Steunenberg (1992) establishes whether or not a prediction made by a coalition formation theory is better than a random choice out of all winning coalitions. It is based on the goal of precision, where in the ideal case the solution set only contains the coalition that was actually formed. Let \(K\) be the set of cases, which in this text comprises the 28 countries listed in table 2. The number of coalitions in the solution set for country \(k\) is noted as \(|S_k|\), where \(S_k\) is the general notation for a solution set of country \(k\) without specifying the model or method that was used. The number of winning coalitions for the corresponding country is noted as \(|W_k|\). This allows us to define the probability of a correct prediction in country \(k\) as \(z_k = \frac{|S_k|}{|W_k|}\).

The sum of \(z_k\) over all the cases \(k \in K\) is compared to the number of correct predictions by each model or method, calculated as \(\sum_{k \in K} c_k\), where \(c_k\) is equal to 1 if the historical (actual) coalition was an element of \(S_k\) or 0 if the prediction was not correct. The number of correct predictions for the power oriented theories are found at the bottom of table 2. The results for the spatial models are featured in table 8. The number of correct predictions varies between 1 and 13 out of 28 countries. Although these values do not convey the entire picture, there seems some evidence that the minimal distance variance model is outperformed by the other three solution methods. The three remaining methods achieve roughly the same values, with some minor variation. At first glance, there appears little difference between the applications with different distance functions. Also, whether or not saliences are allowed to vary (weighted or unweighted) seems to have little effect on the number of correct predictions.

Based on the probability of selecting a correct coalition, Steunenberg (1992) formulated the following null- and alternative hypothesis that can be tested for each theory:
Table 8 Number of correct predictions for the spatial solution methods.

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Euclidean</th>
<th>Sq. Euclidean</th>
<th>Rectangular</th>
<th>Classical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal aggregated distance</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Minimal distance variance</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Maximal satisfaction</td>
<td>11</td>
<td>11</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>Maximin satisfaction</td>
<td>10</td>
<td>10</td>
<td>13</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 9 Z-scores for the power oriented models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Z-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal winning</td>
<td>7.63**</td>
</tr>
<tr>
<td>Minimal size</td>
<td>4.52**</td>
</tr>
<tr>
<td>Minimum number</td>
<td>4.68**</td>
</tr>
</tbody>
</table>

Note: ** Significant at the 1% level, * significant at the 5% level

\[
\begin{align*}
H_0 : \sum_{k \in K} c_k & \leq \sum_{k \in K} z_k \\
H_1 : \sum_{k \in K} c_k & > \sum_{k \in K} z_k
\end{align*}
\]

Not being able to reject the null-hypothesis means that the specific theory is less precise than the random selection of a coalition. Therefore, we expect to reject the null-hypothesis and thus accept the alternative hypothesis. Using the central-limit theorem, Steunenberg (1992) derives the normally distributed test statistic \(Z\) that is computed as

\[
Z = \frac{\sum_{k \in K} c_k - \sum_{k \in K} z_k}{\sqrt{\sum_{k \in K} z_k (1 - z_k)}}
\]

Computing this Z-statistic for the naive example where \(s_k = w_k\) gives a result of 0, compliant with the null-hypothesis and preventing the critique that was mentioned above. The level of significance can be obtained from the normal table, which makes the test easy to apply (Steunenberg, 1992). Table 9 shows both the descriptive results as well as the significance level for the power oriented model. The first observation is that all three models perform significantly better, at the 1% level. Secondly, the minimal winning solution has the highest Z-score out of the three meaning that its performance is the best. It implies that the larger number of coalitions in the solution set is more than compensated by a higher number of correct predictions.

The Z-scores from table 9 now serve as the benchmark for the performance of the spatial models. The Z-scores and their level of significance for the various combinations of the solution methods, distance functions and issue saliences is featured in table 10. Looking at the Z-scores, we notice that many of the combinations of the factors have a descriptive value that is higher than the Z-score of the minimal winning model. All possible applications of the minimal aggregated distance method score higher than the minimal winning model. This is also the case for nearly all configurations of the maximal and maximin satisfaction methods. Each of both methods have one application that scores less than the minimal winning model, but these cases still outperform the minimal size and minimum number models from table 9. In any case, all configurations of these three methods are significantly better than the random choice at the 1% level.

The more worrying case is the minimal distance variance method. Apart from one (salience-unweighted rectangular distance), all applications of this method have a lower Z-score than the models in table 9. This means that although the method uses much more information to make a prediction, it does not perform better than the relatively simple power-oriented theories. Most applications perform significantly better than the random choice, three at the 1% level and another three at the 5% level. But there are two applications that do not even meet this absolute lowest standard of performance. When the minimal distance variance
Table 10 Z-scores for the spatial solution methods.

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Euclidean</th>
<th>Sq. Euclidean</th>
<th>Rectangular</th>
<th>Classical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal aggregated distance</td>
<td>10.24**</td>
<td>10.24**</td>
<td>9.29**</td>
<td>8.34**</td>
</tr>
<tr>
<td>Minimal distance variance</td>
<td>3.58**</td>
<td>3.58**</td>
<td>1.67*</td>
<td>1.67*</td>
</tr>
<tr>
<td>Maximal satisfaction</td>
<td>9.29**</td>
<td>9.29**</td>
<td>7.39**</td>
<td>6.43**</td>
</tr>
<tr>
<td>Maximin satisfaction</td>
<td>8.34**</td>
<td>7.39**</td>
<td>11.19**</td>
<td>7.39**</td>
</tr>
</tbody>
</table>

Note: ** Significant at the 1% level, * significant at the 5% level, Unw.= unweighted with saliences; Wei.= weighted with saliences.

is used with the classical application, either salience-weighted or -unweighted, there is a negative Z-score. The classical application means that we use the Euclidean distance function in combination with the gravity center for consensus estimation, as explained in section 2.3.3. The salience-unweighted case corresponds to the original definition of the method by Boekhoorn et al (2006). Finding that their method yields a lower Z-score than a random choice of coalition already raises some serious questions regarding its predictive power.

From this section, we know that there is a noticeable difference in the predictive performance of the power oriented models and some of the spatial models or configurations. There is also a marked difference in performance between the spatial models themselves. Some of them appear to make good predictions, at least better than random choice and power oriented models, while others deliver poor results. The limitation of the Steunenberg (1992) test is that we can only compare statistically against the random choice. The comparison between the different models is solely based on the descriptive results of the Z-score.

3.3.2 Which factors affect the predictive power

We have presented three factors that can potentially have an impact on the probability of predicting the correct coalition: the solution method, the distance function and the possible variation in issue saliences. We already got an indication in the previous section that the minimal distance variance method might be less suitable than the others. To determine precisely which of these factors, if not all, have an impact on the predictive ability we will use a three way repeated measures ANOVA. Three way because there are three explanatory factors and repeated measures because the same 28 countries are used for every combination of the factors’ values.

Determining the explanatory variables for the ANOVA is straightforward. Each of the three factors is an independent variable. However, choosing an appropriate dependent variable for the ANOVA is less clear. Unlike the independent variable which can be a categorical variable, the dependent variable has to be measured on interval scale. We will apply the ANOVA twice, each time with a different dependent variable.

The first dependent variable is the rank percentile. It gives the relative place of the historical coalition that was actually formed in the ranking of all winning coalitions for the given country. It is calculated as

\[
\frac{|W_k| - \rho_k + 1}{|W_k|} \times 100
\]

(23)

where \(|W_k|\) is the number of winning coalitions in country \(k\) and \(\rho_k\) the rank of country \(k\)’s historical coalition. Let us take the example of Belgium after the national elections of 2003 (Rihoux et al, 2004). Based on the electoral results we can determine that \(|W_k| = 502\). Let us suppose now that we use the minimal aggregated distance method to rank these 502 potential coalitions. The coalition with the lowest aggregated distance receives rank 1 and the one with the highest value is ranked 502nd. The historical Verhofstadt II government is ranked as number 160 if we take the results of the consistent salience-unweighted Euclidean distance application. This gives a rank percentile of 68.33 for this combination of the factors. If the application predicts the historical coalition, e.g. in countries like Australia and Germany, the rank percentile will be 100. Obviously, the ranking procedure is reversed when the solution method aims to maximize a criterion. For example, the highest satisfaction score will receive the rank 1 and the coalition with the
Table 11  Average rank percentile for the spatial solution methods.

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Euclidean</th>
<th>Sq. Euclidean</th>
<th>Rectangular</th>
<th>Classical</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal aggregated distance</td>
<td>94.45</td>
<td>94.45</td>
<td>93.00</td>
<td>93.47</td>
<td>93.51</td>
</tr>
<tr>
<td>Minimal distance variance</td>
<td>63.01</td>
<td>63.74</td>
<td>76.37</td>
<td>77.98</td>
<td>66.04</td>
</tr>
<tr>
<td>Maximal satisfaction</td>
<td>91.14</td>
<td>91.07</td>
<td>87.89</td>
<td>87.88</td>
<td>90.30</td>
</tr>
<tr>
<td>Average</td>
<td>82.80</td>
<td>83.85</td>
<td>80.15</td>
<td></td>
<td>82.15</td>
</tr>
</tbody>
</table>

Table 12  Average percentage of optimality for the spatial solution methods.

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Euclidean</th>
<th>Sq. Euclidean</th>
<th>Rectangular</th>
<th>Classical</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal aggregated distance</td>
<td>83.94</td>
<td>83.52</td>
<td>77.72</td>
<td>77.00</td>
<td>83.75</td>
</tr>
<tr>
<td>Minimal distance variance</td>
<td>45.53</td>
<td>47.71</td>
<td>23.84</td>
<td>26.00</td>
<td>44.32</td>
</tr>
<tr>
<td>Maximal satisfaction</td>
<td>91.42</td>
<td>91.60</td>
<td>92.88</td>
<td>93.01</td>
<td>92.21</td>
</tr>
<tr>
<td>Average</td>
<td>68.30</td>
<td>65.83</td>
<td>67.09</td>
<td></td>
<td>64.56</td>
</tr>
</tbody>
</table>

Our second dependent variable is the percentage of optimality. It is not based on the ranking, but rather on the winning coalitions’ scores for the criterion that is used in the respective solution methods. This means that the calculation of this variable depends on the solution method for which it is applied, more precisely whether the goal is the maximization or the minimization of a criterion. Let us start with the two minimization methods: aggregated distance and distance variance. Again, we use the case of Belgium for the minimal aggregated distance method with the consistent salience-unweighted Euclidean distance application. The prediction by the model is a coalition of Christian democrats and liberal democrats, the Flemish speaking and the French speaking parties of both ideologies. This prediction will be designed as the theoretically optimal coalition ($A_{Opt}$), which has the minimal aggregated distance of $\text{Agg}(A_{Opt})=52.25$. Evidently, this is less that the result of the historical coalition ($A_{His}$) with $\text{Agg}(A_{His})=96.07$. Dividing both scores and multiplying by 100 gives a percentage of optimality equal to 54.39. Now let us translate this procedure to the maximization methods: maximal satisfaction and maximin satisfaction. We use the same case as before, but substitute the minimal aggregated distance with the maximal satisfaction method. By chance, the theoretically optimal coalition is also the same as before but now with an optimal result of $\text{Sat}_1(A_{Opt})=0.77$. The historical coalition scored $\text{Sat}_1(A_{His})=0.70$. Dividing these values in the same manner as before would not give a comparable measure here, since the values in the minimization case are limited between 0 and 100. To assure that this limitation holds here as well, we divide the $\text{Sat}_1(A_{His})$ by $\text{Sat}_1(A_{Opt})$ giving a percentage of optimality of 90.85. Table 12 gives the average percentage of optimality for every combination of the factors.

Comparing the descriptive results in tables 11 and 12 can give an indication of where to look for the significant effects with the help of the ANOVA. Starting with the factor solution method, we notice that the ranking of the methods in terms of performances changes with the choice of dependent variable. The minimal aggregated distance method gained the highest average rank percentiles, followed by the maximal satisfaction solution method. This order is reversed if the predictive performance is measured by the percentage of optimality. In line with the results from the previous section, the minimal distance variance method has the lowest average prediction rates, irrespective of the dependent variable. This leaves the maximin satisfaction method in third place for both dependent variables.

The effect of the second factor distance function is less clear. Based on the average rank percentile per distance function, the consistent squared Euclidean distance application performs best, followed by the classical application and the consistent rectangular distance application in last place. This is not the case for the percentage of optimality, that gives the same ranking as the order in which they are featured in table 12.
The Euclidean distance gives the highest average percentage, and the classical application is the worst case. The third factor of issue saliences appears to have the least effect on the predictive power, no matter the choice of dependent variable. The average rank percentile for the salience-unweighted applications is 81.92 versus 82.56 for the salience-weighted cases. This order is reversed for the percentage of optimality where the salience-weighted cases (66,36) have a slightly lower average than those that are salience-unweighted (66,53). However, in both comparisons the difference is relatively small.

This leads us to the three-way repeated measures ANOVA to establish whether these observed differences in averages are an indication of a significant impact by the factors on the ability to predict the correct coalition. This analysis is based on the textbook by Field (2010) and was performed using IBM SPSS Statistics 20. Before we can go ahead with the main analysis, we need to address the assumption of sphericity (denoted by ε and sometimes referred to as circularity) which refers to the equality of variances of differences between different combinations of factor settings (Field, 2010). In SPSS, it is examined by Mauchly’s test. For the rank percentile, the test indicated that the assumption of sphericity was violated for the factors solution method (χ²(5)=32,088; p < 0.001) and distance function (χ²(5)=30,567; p < 0.001) as well as for all interaction effects. Therefore, degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity (ε=0.548 for the main effect of solution method and 0.635 for the main effect of distance function). These findings are mirrored when using the percentage of optimality as the dependent variable. The assumption of sphericity was violated for the factors solution method (χ²(5)=28,024; p < 0.001) and distance function (χ²(5)=20,274; p < 0.05), in addition to all of the interaction effects. The values of the Greenhouse-Geisser correction are the same as for the rank percentile. No p-value is given for the factor issue salience since “you need at least three conditions for sphericity to be an issue” (Field, 2010). This implies that we can determine the impact of this under the assumption of sphericity, without any correction.

Confirming the descriptive results from table 11, we find a significant main effect of the choice of solution method on the average rank percentile (F(2,030;50,743)=15,562; p < 0.001). From the post-hoc pairwise comparisons, we conclude that both the minimal aggregated distance method performs significantly better than the minimal distance variance (p < 0.001) and maximin satisfaction methods (p < 0.05). There also exists a significant difference between the maximal satisfaction and the minimal distance variance methods (p < 0.001). Similarly, there is a significant main effect of the factor solution method when the percentage of optimality is the dependent variable (F(1,643;37,792)=36,855; p < 0.001). However, contrary to the previous case, all pairwise comparisons of solution methods resulted in significant differences in average percentage of optimality.

Using the rank percentile as dependent variable, there appears no significant main effect by the factor distance selection (F(1,601;40,034)=1,917; p > 0.05). This means that, when the values for the other two factors are not known, the choice of distance function has no significant impact on the average rank percentile of the historical coalition. However, there is a significant interaction effect of the solution method and the distance function on the average rank percentile (F(3,887;97,187)=3,916; p < 0.05). These results were confirmed when the ANOVA was performed with the percentage of optimality as dependent variable, no significant main effect (F(1,906;43,841)=1,917; p > 0.05) but a significant interaction effect of solution method and distance function (F(3,786;87,085)=9,472; p < 0.05).

The choice of distance function can have a significant effect, but only when certain solution methods are applied. And we did find such significant effect when performing a two-way repeated measures ANOVA, solely using the results for the minimal distance variance method (F(1,544;41,693)=5,034; p < 0.05). Given this solution method, the consistent squared Euclidean distance application performs significantly better than the classical application, the consistent Euclidean and rectangular distance applications respectively. The difference between the latter three possibilities are not significant at the 0.05 level. This story becomes quite different when we look at the percentage of optimality variable. Two-way repeated measures ANOVA’s for the results of all four solution methods separately indicated that the factor distance had a significant impact on the average percentage of optimality in each of them. The obvious question is then
why this factor has no significant impact when we analyze all these results together. The reason is that the relative performance of the distance function is different for every solution method, the rankings are completely scrambled compared to one another. This means that when they are all put together in one dataset, the differences will cancel each other out making the effect difficult or impossible to find in a general analysis.

The final factor we examined is the difference between salience-unweighted and salience-weighted solutions. In the first case, all calculations are made under the assumption that \( s_{ip} = \frac{1}{n_i} \) for all \( i \in I \) and all \( p \in P \). In the salience-weighted case, we relied on empirical estimations for the relative importance that parties attributed to the various policy domains that fall under the responsibility of the future coalition. Intuitively, one might expect that allowing the issue saliences to vary across parties and across dimensions would have a clear positive impact on the correct prediction of the coalition formation. The inclusion of additional variables and thus information should have that effect. Surprisingly, this is not what we found in our ANOVA results. Neither for the rank percentile (\( F(1;25)=0.665, p > 0.05 \)) nor for the percentage of optimality (\( F(1;23)=0.097, p > 0.05 \)) did we find a significant impact of the issue saliences. This means that there is no significant difference between the results that were calculated salience-unweighted and salience-weighted.

Because the two-way interaction effects between distance and solution method also did show significant effects, we turned to the three-way interaction effect of solution method, distance function and issue saliences on the average rank percentile. This implies that we first need to determine combinations of a solution method and distance function, for which we then compare the salience-unweighted and salience-weighted results. To get started, we chose the largest difference from table 11 as this was the most likely candidate. Our suspicion was confirmed by the significant effect of issue saliences (\( F(1;27)=5,528, p < 0.05 \)), when only applying the minimal distance variance method with rectangular distance. The problem with this finding is that the effect is in the wrong direction, the salience-unweighted application is significantly better than the salience-weighted. The second possible case was the minimal distance variance method with the classical application, where the effect was in the expected direction but not significant (\( F(1;27)=4,072, p > 0.05 \)), although a border case with \( p=0.054 \). This means that no other combination of solution method and distance function could generate a significant effect of issue saliences. The use of the percentage of optimality as dependent variable resulted in the same findings.

One potential explanation for the absence of the effect of issue saliences could be the lack of variation in the original data. When the saliences that are used in the salience-weighted case do not differ significantly from \( \frac{1}{n_i} \) it would be only logical that there is little impact on the predicted coalitions. This why we calculated for each of the 28 countries the coefficient of variation of the \( s_{ip} \) that were used in the case study. The observed coefficients of variation ranged from 0.1233 to 0.3926. These values are relatively low, but at least in all countries there is some variation in the empirical issue saliences. For example, we did not come by any party in any country for which the expert survey by Benoit and Laver (2006) indicated that there was no variation at all from \( \frac{1}{n_i} \). Every party was attributed with some priority issues. Based on the statistical test devised by Abdi (2010), we also found that the coefficients of variation for all countries were significantly larger than 0. This reduces the probability that the lack of variation could be the sole explanation for this noteworthy finding regarding issue salience, although it can be part of it.

Obviously, the discussion up to this point has been made under the assumption that issue saliences should have an impact on the electoral results. An alternative explanation, for the lack of evidence for the positive impact of allowing issue saliences to differ between policy dimensions and parties on the ability to predict the correct coalition, could simply be that there is no impact. However, it is impossible to conclusively determine whether or not issue saliences play an important role in the coalition formation process based on the present analysis. These results are only based on a case study comprising 28 countries, with only one electoral result included for each country. Some caution is in order when interpreting them. But certainly further empirical confirmation could provide additional insights in the role these factors play in spatial coalition formation models. For instance, the role of issue saliences could be greater if they have a
more prominent role in alternative formation models. In these solution methods presented here, the issue saliences only influence the results indirectly as part of the calculation of the distances.

4 Conclusions and further research

The main goal of this analysis was to establish the impact of three factors on the ability or probability of a model to predict the correct coalition. To establish which of the factors has a significant impact, we performed an empirical investigation based on 28 democracies, member states of either the Organization for Economic Co-operation and Development (OECD), the European Union or both. One national election and subsequent coalition formation was included for each country. All included elections took place around the period 2002-2004, in line with the empirical estimation of the issue saliences and positions. The first factor we examined was the solution method used to predict the coalition that will be formed. Both the descriptive results as well as the statistical analysis showed that the criterion or method used to rank the set of winning coalitions has a significant impact on the average performance of the model, measured as the rank percentile or percentage of optimality. Depending on the criterion, either the minimal aggregated distance or the maximal satisfaction solution method scored best. The minimal distance variance method gave the poorest predictions, irrespective of the used measure. The maximin satisfaction solution method ranked third in both comparisons.

The conclusions for the second factor, distance selection, are not as clear cut. We found no main effect by the choice of distance function on the rank percentile. Only when we restricted the analysis to the minimal distance variance method did we find a significant impact, where the squared Euclidean distance outperforms the remaining possibilities. Neither did we find a main effect of the distance function on the average percentage of optimality, but separate analyses for each solution method revealed another picture. The distance function has a significant impact when applied to each solution method independently. But the order of performance is different for each solution method.

Another (often forgotten) variable that could influence coalition formations are issue saliences. These saliences are a measure of the importance that parties attribute to the relevant issue dimensions (policy domains that fall under the responsibility of the coalition). Until recently, most spatial coalition formation models assumed that all issue dimensions are equally important, and that is the case for all parties. Strangely enough, our results indicated that whether or not issue saliences are allowed to vary there is no significant effect on the prediction of the correct coalition. On the contrary, the few significant results would lead us to believe that salience-weighted results perform worse than salience-unweighted. But these are only isolated examples. The most applications showed that there is little or no difference between both options. Additional empirical testing would certainly be appropriate, before concluding that previous authors who excluded issue saliences from their models all together were right to do so. Only one electoral result from each of the 28 countries was used in the present analysis, so caution should be taken before extrapolating the conclusions. Including issue saliences in other coalition formation models or using other case studies could yield very different outcomes.

Our results also indicated that the (significant) effects found can depend on the dependent variable that was used, either rank percentile or percentage of optimality. This should certainly be taken into account when choosing the appropriate combination of solution methods, distance function and/or the inclusion of issue saliences. It was not our goal to select one or the other dependent variable as the most suitable, that is why we presented both as each has its merits and limitations. Finally, we present some additional avenues for further research.

– It could be interesting to test the model in a business setting. Other applications may include the negotiations of international treaties like the Doha Round of the WTO. The main issue with these applications is the lack of information necessary for the use of the model. To our knowledge, no surveys or studies have defined relevant issue dimensions in these setting, let alone the issue positions and saliences.
In section 2.3.1, we already briefly mentioned that the present analysis is based on the sum-1 normalization of issue saliences. Another factor that could influence the predictive power of the coalition formation models is the use of alternative normalizations such as the product-1 or the sum of squares. In theory, all configurations that are featured here could be applied with these two alternatives. However, this would require alternative sources of empirical estimations which are in line with these definitions.

The assumption in the present work of symmetric distances could be abandoned, allowing the use of asymmetric distances. This would imply that changes in location along an issue dimension in different directions could be valued differently by the parties.

Another generalization of the model could be the implementation of non-separable issue saliences, the so-called interaction effects. The main problem again will be the lack of information such as empirical estimations of the interaction effects.

Apart from the limitations that parties may have on the location of the consensus position, they can also have reservation on the partners with whom they form the coalition. This can be because of historical or ideological reasons. At this point, these impossible combinations of parties must be filtered out of the set of possible solutions on an ad hoc basis. It could be useful to examine ways in which this could be done automatically within the scope of the model.

An alternative way of defining the consensus point could be by choosing the point that minimizes the variance of the distances between itself and the parties’ ideal positions, rather than the sum of distances. This estimation method would be more in line with the minimal distance variance solution, which was presented in section 3.2.2.

The optimal solution is chosen in the various solution methods as the coalition with the best score on the appropriate criterion. This score is calculated for the coalition as a whole. However, a situation could arise where a member of the optimal coalition would be better of being part of another coalition, which is not the optimal solution. Considering the possibility that a party can change from one coalition to another based on its individual preferences could be an interesting direction for further research. This would require an extensive analysis on the stability of the coalitions. An overview of various definitions of coalition stability has been presented by Hajdukova (2006).

We have included only four solution methods in the present analysis. It could be interesting to develop variations of these solutions methods, using different ways to rank all possible winning coalitions.

References

Axelrod R (1970) Conflict of Interest; A Theory of Divergent Goals with Applications to Politics. Markham, Chicago
Issue saliences and distance selection in spatial coalition formation models: an empirical investigation


Notes
1 The reader may find that this definition differs from the original formulation of Kohlberg (1971) in the sense that we have substituted his notation with ours. This is also the case for all other definitions that follow.
2 The reader should note that the survey was performed during the period 2002-2004. This means that the majority of responses was gathered before the 2004 and 2007 European Union enlargements.
3 We have chosen not to include Albania in our case study at this time. The closest election to the survey results took place in 2001, but was described as “flawed” (Fischer, 2010). The next parliamentary elections took place in July 2005 with improved reliability compared to 2001. But by 2005 several parties included in the survey of Benoit and Laver (2006) had merged into cartels and new parties had been founded. This restricts the usability of the survey as there are no estimations for issue positions and saliences for these new parties and cartels.
4 We have chosen not to include Belarus in our case study since, as mentioned by Benoit and Laver (2006), “most parties did not participate in the 2003 election, and since President Aleksandr Lukashenko’s government is not formed from a party”, in addition to the fact that “most MP’s are independents”. The parties, whose issue positions and saliences were surveyed, had very little, if any, impact in the government formation process.
5 We have chosen not to include Ukraine in our case study because of the volatile political situation around the period of the survey. The period around the 2002 parliamentary election preceded the Orange Revolution. Also the government was not backed by a parliamentary majority, it was supported by various independent MP’s. The positions of these individual MP’s were not included in the survey.
6 For the United Kingdom, Benoit and Laver (2006) made the distinction between Britain and Northern Ireland. However, the United Kingdom is not relevant for the case study of our model. In both the national elections for the House of Commons which are closest to the survey period, on June 7th 2001 (Fisher, 2002) and on May 5th 2005 (Fisher, 2006), the Labour party gained an absolute majority. This allowed the formation of the Blair II and III governments without the necessity of coalition formation.
7 The vote shares provided by Benoit and Laver (2006) were the result of the election to the House of Commons which was held on November 27th 2000 (Carty, 2001). However, this election resulted in a majority of seats for the Liberal Party. As there was no need to form a coalition the case is not useful for the model.
8 We shall not include France in our case study because of the fact that the Union pour la majorité présidentielle (UMP) won an absolute majority of seats in the Assemblée nationale in the election of June 9th 2002 (Ysmal, 2003). Although they could have...
formed a government without any partner, they choose to form a coalition with the Union pour la Démocratie française (UDF). This is due to the fact that the UMP was both an electoral alliance and the tentative foundation of an integrated moderate right-wing party and "only those who had already decided not to join the UMP are labelled as UDF" (Ysmal, 2005). Also the parties surveyed by Benoit and Laver (2006) do not correspond to those who participated in the 2002 election. An alternative would be to use the results of the 1997 election, where all participating parties were surveyed. But we feel that the period between this election and the survey is too long, making estimation unreliable.

We have not included Greece in our case study because in the elections of March 7th 2004 the New Democracy (ND) party gained an absolute majority of 165 seats out of 300 in the Vouli (Mavrogordatos, 2005). This allowed them to make a government without any coalition partner. The closest election before the survey was held took place on April 9th 2000. This lead to a majority of 158 seats for the Pan-Hellenic Socialist Party (PASOK), who also formed a government without the need to form a coalition (Mavrogordatos, 2001). We feel that other elections are too far removed from the period of the survey.

Malta was not suitable for our case study because the Nationalist Party won a absolute majority of 35 seats out of a total 65 in the national election for the House of Representatives on April 12th 2003 (Fenech, 2004). This enabled Eddie Fenech Adami to form a cabinet without a coalition partner.

Moldova will not be included in our case study since the Party of Communists of the Republic of Moldova won an absolute majority of the seats in the national parliament, in both the elections of February 25th 2001 and of March 6th 2005. This allowed them to form a cabinet without any coalition partner. We feel that other elections are too far removed from the period of the survey too produce reliable results.

We will not include Russia in our case study because the United Russia (UR) party was able to form the cabinet Fradkov II without any other coalition partner. Although the UR caucus in the Duma only had 221 members immediately after the national elections of December 7th 2003, it "was increased to some 310 in early 2004 by the addition of the majority of ‘independent’ deputies" (Sakwa, 2005). This provided them with an absolute majority, given the total number of seats of 450.

We shall not include Spain in our case study after because the national elections of March 14th 2004, that are closest to the period of the survey, the Socialist Workers Party formed the Zapatero I cabinet without a coalition partner although it did not win an absolute majority of seats in Congreso de los Diputados (Delgado and Lopez Nieto, 2005). As this was a minority government, it needed support by other parties but "their legislative support for the term was not public" (Delgado and Lopez Nieto, 2005) although it most likely came from "the ERC (a republican and nationalist radical leftwing party) and IU (a coalition of the Spanish Communist Party, greens and other leftist groups)". These parties did not from a coalition, in line with the tradition of parliamentary support, as opposed to coalitions of Spanish governments "when no party has an absolute majority" (Delgado and Lopez Nieto, 2005). Predicting the group of parties that support the government rather that the formal coalition, like we have done for other countries with minority governments such as Denmark and Sweden, is not possible for Spain. This is because the ERC was not included in the survey by Benoit and Laver (2006). As this party supplied critical seats for reaching a majority, the model can never predict the correct group without it.

Turkey will not be included in our case study because the Justice and Development Party (AKP) won an absolute majority in the parliamentary elections of 2002 and also in the following elections of 2007. This allowed Abdullah G¨’l and Recep Tayyip Erdogan, respectively, to form a government without any coalition partner. We do not use the results from the 1999 elections, were no party gained an absolute majority. The first reason is that not all parties the participate in these elections were included in the survey. Most notably, the Virtue Party was disbanded between 1999 and the time of the survey. As they represented 111 parliamentary seats out of the total 550, they were quite important. Secondly, the period between the 1999 elections and the period of the elections is too large for the estimated issue positions and saliences to be reliable. Both the aspect place serious doubts on the validity of combining the survey with the seat distribution of the 1999 elections. We have rather chosen not the incorporate Turkey as part the the case study.

In line with the vote share provided by Benoit and Laver (2006), the seat distribution in the national Chamber of Deputies after the elections of November 26th 2000 (Mungiu-Pippidi, 2001) would be preferred to use in our case study. The results of the elections of 2004 are not applicable because by then several alliances had been formed, reducing the reliability of the estimations from the survey. But after the elections, the designate prime minister Adrian Nastase explicitly stated "that he would not seek any political alliance but rather govern as a minority government" (Mungiu-Pippidi, 2001). The former government coalition gained "only a quarter of the seats", and subsequently "agreed to support the Nastase government, provided that it stood by the National Strategy for European Integration" (Mungiu-Pippidi, 2001). While governing however, the minority government relied on the support of varying partners, depending on the subject.

We have chosen not to include Bosnia and Herzegovina in our case study because "governing coalitions are not based on a convergence of parties which favor similar policies" (Bieber, 2010). The political organization in the former Yugoslavia countries is largely determined by ethnic problems, which interfere with the coalition formation process when working within the framework of policy seeking theories. Our model does not take the specific balances into account which are required in that region.

We shall not include Northern Ireland as part of our case study because the Northern Ireland Assembly was suspended during the period of the survey by Benoit and Laver (2006). This inhibited the formation of the so-called Executive, with is the regional government. Also, the "minister comprising the Executive are appointed by the main parties using the rather complicated d’Hondt formula to decide their allocation" (Hazleton, 2004). This means that all major parties can join the government if they choose to do so. This implies that the government formation in Northern Ireland falls outside of the scope of policy oriented theories.

We will not include Switzerland in our case study because of the so-called “magic formula” (Hardmeier, 2004). This formula, although broken after the elections of October 19th 2003, states that the government consists of the four largest parties, with a predetermined number of members in the Federal Council. This division of membership in the Federal Council was altered after the
electoral victory of the Swiss People’s Party, but the composition of the government remained the same. Together, the four government parties take up 171 out of 200 seats in the National Council (lower house) and all seats in Council of States (upper house) (Hardmeier, 2004). This leads us to conclude that the government composition is determined by the preference for continuity, rather than electoral results and party policies. This excludes it from the scope of our model.

As is the case with other former Yugoslav countries certain ethnic balances have to be taken into account in Croatian politics. One of the consequences of the ethnic separation is the reservation of a number of parliamentary seats for some minorities. This affects the coalition formation process and the power distribution in parliament. After the 2003 parliamentary election, a minority coalition was formed by Ivo Sanader. This coalition consisted of his own HDZ Party together with two junior parties, the HSLS and DC. The problem of using Croatia as a case study for our model lies with the supporting Independent Democratic Serb Party (ISDP), which won all 3 seats reserved for the Serb minority. This supported was needed for the approval of the minority coalition in parliament. Our problem is the fact that the ISDP is not included in the survey by Benoit and Laver (2006). The combination of being a minority government, which is strictly speaking not possible in our model, and the absence of information for an important partner seriously limits the quality and relevance of our predictions. This leaves no option but to exclude Croatia from our case study.

Macedonia will not be included in our survey because of the fact that the not all parties that won seats in the election for the national Assembly on September 15th 2002 were included in the survey by Benoit and Laver (2006). In other countries this would be corrected by adjusting the percentage required for a majority. The problem here is that the minority parties (e.g. Democratic Party of Turks and Democratic League of Bosnians) that were excluded from the survey provided the crucial seats for the Together for Macedonia coalition they formed together with the Social Democratic Union of Macedonia (SDSM) and the Liberal Democratic Party (LDP). This means that with the available data the model can never predict the correct coalition.

We will not include Serbia in our case study because the G17 party was not included in the survey by Benoit and Laver (2006). As this party provided a crucial number of parliamentary seats to the coalition that was formed after the elections, it is impossible to predict the correct coalition without it.

We use the same case as for example 4. Since there were 3 independent House members, whose issue positions and saliences are not part of the survey, we must apply a correction on the percentage of seats needed for a majority coalition. As the government requires 76 out of 150 seats, and we have included only 147 seats in our case study, we set the required percentage for a majority at 52%.

The same information is used as for example 3.

Corresponding to the vote share given by Benoit and Laver (2006), we employ the results of the federal elections which took place on May 18th of 2003. As described by Rihoux et al. (2004), all but one of the parties whose positions and saliences were estimated won seats in the Chamber of Representatives. The exception being Agalev, the green party of the Flemish Community, which was wiped out of the federal parliament after a scandal. Because of this they were not included in our case study. The Verhofstadt II cabinet was formed as a coalition of the liberal and socialist parties from both the French-speaking and Flemish-speaking regions. These parties are the Flemish Liberal Democrats (VLD), the Reform Movement (MR), the Socialist Party-Different (Sp.a) and the French-speaking Socialist Party (PS). This was a continuation of the previous government also headed by Guy Verhofstadt, with the difference that the green parties from both language communities were excluded due to their massive loss in the elections.

Unlike Benoit and Laver (2006) who present the Bulgarian national parties’ vote shares “based on a poll taken in May 2003”, we choose the use the parliamentary seats obtained in the 2001 election. After that election only four parties were represented in the national parliament (Spirova, 2010). Another option would have been to use the results of the 2005 national election, but by that time several new parties and alliances (e.g. the National Union Attack) had been founded which were not part of the survey. Following the elections the National Movement for Simeon II (NDS) and Movement for Rights and Freedom (DPS) supported the first cabinet that was headed by Simeon Saksoburgotski. The NDS won exactly half of the parliamentary seats in the election, but chose to cooperate with the DPS as they did not have an absolute majority.

Although it was formed before the congressional election of December 11th 2005 (Heath, 2007), we consider the Coalition of Parties for Democracy in our case study as the government coalition. This coalition was made up of the Christian Democrat Party (PDC), the Party for Democracy (PPD), the Socialist Party of Chile (PS) and the Social Democrat Radical Party (PRSD). As some seats are not represented in our case study we need to make some corrections. The survey by Wiesehomeier and Benoit (2009) did not include 4 independent MP’s and the Regionalist Action Party of Chile, which won one seat. This means that only 115 out of the total 120 are represented. The necessary percentage for a governmental majority will be set at 53%, which corresponds to 61 seats out of 115.

Important to note is the fact that we restrict our analysis to the Greek part of Cyprus. This choice in not personal, but simply due to the fact that Benoit and Laver (2006) only provide information for the parties of Greek Cyprus. Another specific aspect of Cypriot politics is the fact that the President is both the head of state and the head of government. As a consequence, the party to which the president belongs will inevitably be part of the government, even if it is relatively small in the House of Representatives. This could lead to some discrepancy between the predictions of our model and the real situation. We use the results of the elections that took place on May 27th 2001, i.e. the seats corresponding to the vote share which was provided by Benoit and Laver (2006). The government we want to predict is Papadopoulos I, which was invested on February 28th 2003 as a result of the presidential election that took place earlier that month (Ker-Lindsay and Webb, 2004). It was supported by the Democratic Party (DIKO), the Progressive Party of the Working People (AKEL) and the United Central Democratic Union (EDEK).

We solely consider parties who won seats in the Chamber of Deputies during the national election of June 14th and 15th 2002 (Linek, 2003). This means that not all of the parties that were surveyed by Benoit and Laver (2006) will be used. But since they did not have any parliamentary representation, they had no impact or role in the coalition formation process. All parties with representation in the national parliament were included in survey. Subsequent to the elections the Social Democratic Party (CSSD) and the pre-electoral
coalition of the Christian Democratic Union-Czech People’s Party (KDU-ČSL) and Union of Freedom-Democratic Union (US-DEU) supported to formation of the Špidla I government.

29 As is the case in other Scandinavian countries, Denmark has a tradition of minority governments. The two elections that are closest to the survey results took place on November 20th 2001 (Bille, 2002) and February 8th 2005 (Bille, 2006). Both resulted in the formation of the minority cabinet Rasmussen I, supported by a coalition of the Liberal Party (V) and the Conservative People’s Party (KF). We assume that the Danish Peoples’ Party (DF) is a de facto member of the coalition. As “the distribution of seats in Parliament gave no other alternative”, both in 2001 and 2005, the “government had to base its policy on support from the Danish Peoples’ Party” (Bille, 2002). We use the seat distribution in the Folketinget after the 2001 election.

30 Corresponding to the vote share provided by Benoit and Laver (2006), we choose the seat distribution in the Riksdag that resulted from the elections of March 2nd 2003 (Pettai, 2004). The six parties with representation in the national parliament were included in the survey. Following this elections, a majority coalition was formed under prime minister Parts. The Parts I cabinet was supported by Res Publica (RP), the Reform Party (RE) and the People’s Union (RL).

31 Corresponding to the vote share provided by Benoit and Laver (2006), we use the seat distribution in the Riksdag that resulted from the election of March 16th 2003 (Sundberg, 2004). All parties with representation in the national parliament were included in the survey. The Vanhanen I cabinet was supported by the majority coalition of the Centre Party (KESK), the Social Democratic Party (SDP) and the Swedish People’s Party (SPP).

32 We use the seat distribution in the Bundestag after the general election of February 22nd 2002. Only 5 parties are relevant to our case study as there are only 6 parties with parliamentary representation, in addition to the fact that the CDU and CSU are regarded as a single party in the survey because of their common positions in federal issues. The various smaller, regional parties included in the survey are irrelevant to the 2002 coalition formation process. The result of those negotiations was the Schroeder II cabinet, supported by the Social Democrats (SPD) and the Alliance 90/Greens (Poguntke, 2003).

33 The case for Hungary is based on the same data as in example 5.

34 In line with the vote shares provided by Benoit and Laver (2006), our case study is based on the seat distribution in the Althingi after the general election of May 10th 2003 (Hardarson and Kristinsson, 2004). We exclude the New Force party, as they did not win any seat in parliament and subsequently did not have any weight in the coalition formation model. Otherwise, all parties with representation in the national parliament are included in the case. Together, the Independence Party (IP) and the Progressive Party (PP) constituted a majority coalition to support the Oddsson IV government.

35 We use the seats distribution in the Dáil Éireann that resulted from the national elections of May 17th 2002 (O’Malley and March, 2003). This was the only national elections that took place during the period of the survey and also corresponds to the vote shares provided by Benoit and Laver (2006). Following this election Fianna Fáil (FF) and Progressive Democrats (PD) together formed the Ahern II coalition government. A small correction needs to be made in the percentage required for a majority, due to the fact that the Socialist Party and several independents were not included in the survey. They won respectively 1 and 13 seats in parliament out of a total 166 (O’Malley and March, 2003). This leaves 152 seats represented in our case study and 84 seats needed for a majority in the complete parliament. We shall therefore set the percentage required at 56%.

36 Because “immediately after the 27 February 2003 elections, Yisrael Ba’Aliyah joined the Likud and ceased to exist as an independent parliamentary fraction” (Diskin, 2004), we have chosen to combine their seats and consider them as a single party. Also, Yisrael Ba’Aliyah was not included in the survey by Benoit and Laver (2006). Apart from this we use the seat distribution in the Knesset that followed from this election. All other parties who gained representation in the Knesset were included in the survey. Subsequently, the Sharon II coalition government was formed with the support of the Likud, Shinui, National Unity and National Religious Party who together controlled a 68 out of 120 majority.

37 Some caution must be given to the choice of party weights which resulted from the election that took place on May 13th 2001. As stated by Ignazzi (2002), “the total number of seats allotted is 616 instead of 630 because 14 in the plurality part have not yet been assigned”. Our calculations showed that only 609 seats were allotted in the list. Also, there exist some discrepancies between the list of parties surveyed by Benoit and Laver (2006) and the parties who won seats in the Camera dei Deputati as listed by Ignazzi (2002). Firstly, the Nuovo PSI won 3 seats, but was not surveyed. Secondly, the Greens and the Italian Democratic Socialist (SDI) formed the Sunflower alliance in the 2001 elections and together won 17 seats, but their positions and saliences were surveyed separately. As the alliance was disbanded soon after the elections, their separate seats will be used in our case study; 8 for the Greens and 9 for SDI. Thirdly, the positions and saliences that were estimated for the Unione di Centro (UDC) in the survey will be used as a proxy for the alliance of the Centro Cristiano Democratico (CCD) and the Cristiani Democratici Uniti (CDU) who together won 40 seats. Fourthly, the Sudtroler Volkspartei and the Lista Valle d’Aosta we not included in the survey although they won 3 and 1 seats in parliament respectively. Finally, three parties that were included in the survey are not used in the case study as they did not win any seats. Combining all these modifications requires us to set the majority percentage at 53%. This corresponds to 316 seats required for a majority in the total 630, out of the 602 that were included in our case study. The Berlusconi II cabinet that was formed after the elections was supported by a coalition of Forza Italia (FI), the National Alliance (AN), the North League (LN) and the UDC.

38 Contrary to what we have done for most countries, we use the seats distribution of the upper house of the bicameral Diet that resulted from the election on November 9th 2003. More specifically, we shall use the situation in the House of Councilors “after merger of the Democratic Party and the Liberal Party on 24 September” (Kato, 2004). The reason for this exception is that the Liberal Democratic Party (LDP) has an absolute majority the House of Representatives (lower house). But because they did not have the same majority in the upper house other parties were needed to from a coalition. We feel that the seat distribution in the House of Councilors better reflects the political reality during the formation of the Koizumi II cabinet. This coalition government was supported by the LDP, the New Conservative Party (NCP) and the Komei Party. As the survey by Benoit and Laver (2006) only includes the six major
for human rights in United Latvia that Benoit and Laver (2006) made separate estimations for the People's Harmony Party (TSP), which took part in the elections as a study.

Parties, the total of 247 seats is not represented in our case study. To adjust for the omission of some smaller parties, we set the percentage required for a majority at 53%. This corresponds to 124 seats out of the 234 that are represented by the parties in the case study.

We use the results of the national election to the Saeima that took place October 5th 2002 (Ikstens, 2003). Noteworthy is the fact that Benoit and Laver (2006) made separate estimations for the People's Harmony Party (TSP), which took part in the elections as a part of the For Human Rights in United Latvia alliance. As we only know the total number of seats for this alliance and not for the individual parties of which it was made, we exclude the TSP as an independent party from our case study. All the parties with seats in the national parliament are included in the case. Following the elections the Repjie I government was formed as a coalition of the New Era party (IL), the Union of Greens and Farmers (ZZS), the First Party of Latvia (LPP) and For Fatherland and Freedom/LNNK (TB/LNNK).

The period during which the survey by Benoit and Laver (2006) took place was a very volatile one in Lithuanian politics. After the elections of October 8th 2000 several alliances were formed and disbanded. This leads to some discrepancies between the parties as they are listed in the survey and the list of parties who have representation in the Seimas as described by (Krupavivius, 2002). We shall use the seat distribution that was in place after January 1st 2004 and which supported Buzzauskas I cabinet. This government was supported by a coalition of the New Union/Social Liberals (NU/SL) and the Lithuanian Social Democratic Party (LSDP). We incorporate the changes in seats mentioned by (Krupavivius, 2004). This list corresponded the best with the list of parties that were surveyed. We shall use the positions and saliences of the Lithuanian Christian Democrats (LCD) in combination with the seats attributed to the Lithuanian Christian Democratic Party (LCD). This latter party was a splinter group from the former party in reaction to the merger with the Christian Democratic Union (CDU). And the saliences were not estimated for the Also the positions and saliences that were estimated for the Union of Peasant and New Democracy Party will be used as a proxy for the Lithuanian Peasants’ Party (LPP). A small correction will be made in the percentage needed for a majority. Because only 127 seats out of a total 141 are represented in the case study, we set the required percentage at 56% which corresponds to 77.

The same data as in example 1 is used for Luxembourg.

We employ the seat distribution in the Tweede Kamer after the elections of January 22nd 2003 (Lucardie and Voerman, 2004). The Balkenende II cabinet was supported by a coalition of the Christian Democratic Appeal (CDA), the People’s Party for Freedom and Democracy (VVD) and Democrats 66 (D66). All parties with representation in the parliament included in the case.

Corresponding to the vote share provided by Benoit and Laver (2006), we use the seat distribution that resulted from the General Election of July 29th 2002 (Vowles, 2003). All seats in parliament are represented by the parties that were included in the survey.

The Alliance Party (AP) was part of the survey but will not be included in the case study as they did not win any parliamentary representation in the election. After the election the Labour Party and Jim Anderton’s Progressive Coalition (PC) formed a minority coalition that supported the formation of the Clark II cabinet. “United Future (UF) quickly agreed to provide support” (Vowles, 2003) for this minority cabinet, while “the Greens could probably be relied on to support the Government’s more left-leaning legislation when United Future might refuse to do so”. This resulted from the fact that “Labour and the Progressive Coalition were in a strong enough position to play off United Future and the Greens against each other in post-election talks” (Vowles, 2003). To adjust our model to this case we assume that United Future is a de facto member of the coalition.

Contrary to Benoit and Laver (2006), who provide the vote percentage in 2001, we use the seat distribution in the Stortinget after the election of September 12th 2005 (Aalberg, 2006). This is the first national election that took place after the period of the survey. Although the Red Election Alliance was part of the survey it will not be considered in our case study as it did not win any seats in parliament. The seven other parties, whose positions and salience were estimated, represent the total 169 seats in the national parliament. After the election the Labour Party, the Socialist Left and the Centre Party formed a majority coalition which lead to the Stoltenberg II cabinet.

Corresponding to Benoit and Laver (2006), who provide the vote percentage in 2001, we use the seat distribution in the Sejm after the election of September 25th 2005 (Jasiwicz and Jasiwicz, 2006). With the exception of the German Minority party, with two seats, all parties that won seats in the parliament were part of the survey. As 458 out of the total 460 seats are represented in the case study we do not need to adjust the percentage required for a majority. In first instance the Law and Justice party (PS) formed the minority cabinet Marcinikiewicz I, which was invested on November 10th 2005. On July 14th 2006 this minority government was supplemented by the Self-Defence party (SRP) and the League of Polish Families (LPR) (Jasiwicz and Jasiwicz, 2007), giving it a majority in parliament. A few days later, it was replaced by the Kaczynski I cabinet that was supported by the same coalition. We will target this latter coalition as our prediction.

We use the seat distribution in the national parliament after the election of March 17th 2002 (Magone, 2003). All parties that won seats were also surveyed by Benoit and Laver (2006), with the exception of the Democratic Unity Coalition. This pre-election alliance between the Communist Party and the Green participated as one entity and won 12 seats. Two seats out of these twelve went to members of the Greens, leaving 10 for the Communists. We shall use this latter division and the weights for the respective parties.

A majority coalition of the Social Democratic Party (PSD) and the Democratic Social Centre/People’s Party (CDS/PP) established the Barroso I cabinet.

Corresponding to the vote share by Benoit and Laver (2006), we use the results of the elections to the National Council of the Slovak Republic that took place on September 20th and 21st 2002 (Ucen, 2003). Some parties that were included in the survey will not be included in our case study as they did win any seats in parliament. The total number 150 seats is represented in our case study. The Dzurinda II cabinet was supported by a coalition of Slovak Democratic and Christian Union (SDKU), the Party of the Hungarian Coalition (SMK), the Christian Democratic Movement (KDH) and the Alliance of a New Citizen (ANO).
48 Corresponding to the vote share by Benoit and Laver (2006), we use the results of the elections to the National Assembly that took place on October 15th 2000 (Fink-Hafner, 2003). All 88 regular parliamentary seats were won by parties included in the survey. Apart from these 88 seats “the National Assembly also includes two seats reserved for the Italian and Hungarian minorities (one seat each)” (Fink-Hafner, 2003). We shall set the percentage required for a majority at 53%, which corresponds to 46 seats out of the 88 included in the survey. After the elections, Drnovšek I government was formed by a coalition of the Liberal Democracy of Slovenia (LDS), the United List of Social Democrats of Slovenia (ZLSD) and the Slovene People’s Party (SLS-SKD). Together they controlled a 54 seat majority.

49 Corresponding to the vote share by Benoit and Laver (2006), we shall use the results of the elections to the Riksdag that took place on September 15th 2002 (Widfeldt, 2003). All 349 seats in parliament were won by parties included in the survey. After the elections the Persson III minority cabinet was formed by the Social Democrats. This cabinet was supported by the “Government bloc, consisting of the Social Democrats (S), Left Party (V) and the Greens (MP)” (Widfeldt, 2003). Our model will try to predict this government bloc, that forms the majority in parliament, although it is strictly speaking not a coalition.

50 As noted by Field (2010), there are other possible corrections for the degrees of freedom. But our results indicated that the choice of correction had no impact on the final conclusions of the model.

51 The results for the pairwise comparisons were calculated with the Bonferroni adjustment for multiple comparisons.